POLYPHASE CURRENTS

THE SPECIALISTS' SERIES

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POLYPHASE CURRENTS

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BY

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"ALTERNATING CURRENTS AND THE THEORY OF TRANSFORMERS" AND
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PREFACE TO SECOND EDITION

In revising this volume for a new edition, the scope and view-point have not been changed. It is not claimed that the treatment of the subject is exhaustive; but an attempt has been made to present the principles underlying the operation of polyphase currents in clear and simple terms. The student whose aim it is to become proficient in the design of polyphase machinery may use the book only as an introduction to more advanced works; and those requiring practical information on such details as switchboard connections, or descriptions of actual machines and apparatus, must seek this elsewhere; but nevertheless the needs of the practical engineer have been constantly in the writer's mind.

The addition of new matter has been deliberately avoided so far as possible; but much of the original matter has been rearranged, rewritten, or entirely omitted. The assumption that the reader has a fair knowledge of continuous currents, but is unfamiliar with alternating currents, is still made, although there is perhaps less reason for it now than at a time when

comparatively young engineers had left the colleges or universities before the subject of alternating currents was adequately taught. The writer believes, however, that the omission of the two introductory chapters would detract from the value of the book.

In the matter of minor alterations, it may be mentioned that almost every figure or diagram has been redrawn; all vector diagrams have been reversed to accord with the internationally agreed convention regarding leading and lagging vectors; and, for the same reason, the symbols used to denote certain physical quantities have been altered.

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PREFACE TO FIRST EDITION

WITH the extended use of polyphase alternating currents for the transmission and distribution of electric power, there would seem to be a demand for a book treating of the theoretical considerations involved in polyphase working in such a manner as to commend itself to practical engineers and those students who are without the mathematical knowledge required for the study of the more advanced works on the subject.

The author has adopted a non-mathematical treatment of the subject throughout, but has made extensive use of graphical methods. The introductory chapters are written with the object of explaining the use of vectors in solving alternating-current problems, and also for the purpose of drawing attention to the chief points of difference between alternating and continuous currents of electricity. It should, therefore, be possible for any student or engineer with a fair knowledge of continuous-current working, but with only a superficial acquaintance with alternating currents, to obtain a thorough grounding in the principles underlying polyphase working, provided he will take the trouble to master the contents of the first two chapters.

The few notes added at the end of the book, in the form of an appendix, deal with certain matters referred to in the text, but which are not essential to the

scheme of the book, or to the elucidation of subsequent chapters.

It may be urged that the subject of electric power transmission has been given undue prominence; but the reason for the somewhat disproportionate length of this section is that the book has not been primarily written for the use of *designers* of polyphase machinery (for these must have a very thorough and detailed knowledge of the subject), but rather for the *user*, or the engineer who has to lay out and to work complete polyphase schemes. If exact details are required of any particular piece of polyphase machinery, this information can always be obtained from the makers of the plant.

Again, Kelvin's law and the principles determining the most economical section of conductors are discussed at some length, not only because they have a very important bearing on the subject of polyphase transmission of power, but also because they do not appear to be generally understood. These questions have been treated in a non-mathematical manner; but the main points at issue have been clearly stated, and the reader should have no difficulty in calculating the most economical size of conductor to suit any given electric transmission scheme.

The author desires to thank the editors of the *Electrical Engineer* and of the *Electrical Review* for the loan of blocks and permission to reprint portions of certain articles that have appeared in these journals.

Ellesmere Park,
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POLYPHASE CURRENTS

CHAPTER I

ELEMENTARY STUDY OF ALTERNATING CURRENTS

As an introduction to the subject of polyphase alternating currents, it will be advisable to consider the general principles which underlie the working of all alternating currents of electricity, whether single-phase or polyphase; and it is proposed to devote this and the following chapter to an elementary study of single-phase currents; the object being to point out the essential differences between direct and alternating currents, and explain the various effects peculiar to alternate-current working.

1. **Definition.**—An alternating current is—as indicated by the name—a current which, instead of being unidirectional, such as the current produced by a battery or a continuous-current dynamo, flows first in one direction and then in the opposite direction in regular succession. In other words, it is a current which, starting from zero value, increases in strength in a *positive* direction until it reaches its maximum positive value; then dies down to zero value, after which it rises to its maximum *negative* value, and again falls to zero; this process being repeated in a periodic manner.

2. **Generation of Alternating Currents.**—There is little difference in principle between an alternator and a direct-current dynamo; it is the absence of the commutator in the alternating-current generator which leads to the current obtained from the terminals or collecting rings being of the character described above. Given any continuous-current generator, it is merely necessary to replace the commutator by a couple of slip-rings connected to suitable points on the armature winding, and to excite the field coils from a separate continuous-current source, in order that currents alternating in direction may be drawn from the armature.

There are many types of alternator which differ little in appearance from multipolar direct-current dynamos, and it will readily be understood that the number of reversals of the current in a given time will depend upon the speed of rotation of the armature and the number of poles in the exciting field.

3. **Graphical Representation of an Alternating Current.**—Let Fig. 1 represent the chart of a centre-zero recording ammeter, such as might be used for registering the charge and discharge currents of a battery. On such a chart, the lapse of time is measured horizontally, from left to right, while the strength of the current is indicated by vertical distances above or below the horizontal centre line. If the distance is measured *above* this horizontal datum line, this is an indication that the current is flowing in a *positive* direction, corresponding, let us say, with a charging current; but if the measurement is made *below* the datum line, this indicates that the current is flowing in a *negative* direction, and that the battery is being discharged.

Referring to the curve in Fig. 1, it will be noticed that, between the times indicated by the points t and t_1 on the

datum line, corresponding to a period of 4 hours and 30 minutes, the charging current has gradually fallen from 50 amperes down to 25 amperes, after which it has increased in strength, only to be entirely interrupted shortly afterwards before gradually increasing again, but this time in a *negative* direction — *i.e.*, as a discharging current.

The variation of an alternating current, both in strength and direction, may be represented in a similar manner.

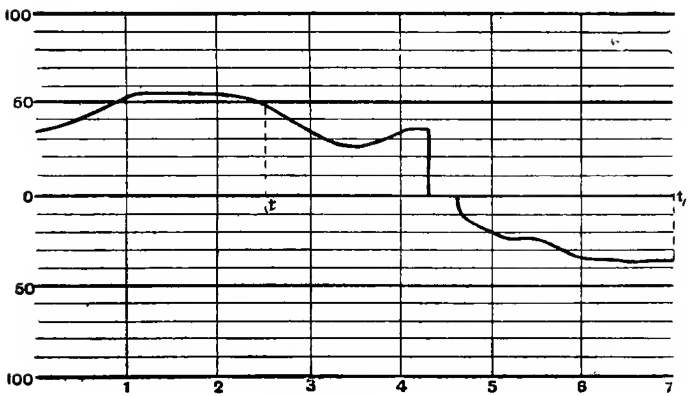

FIG. 1.

In such a curve as that shown in Fig. 2, the lapse of time is measured—as in the previous example—horizontally from left to right, while the strength and direction of the current, at any particular instant, are indicated by the length of the vertical ordinate and its position relatively to the datum line. The chief difference between this and the previous diagram (Fig. 1) lies in the fact that, whereas in the first diagram we were dealing with a lapse of time measured by hours, the total horizontal

distance in Fig. 2 covers only a small fraction of a second of time; and, further, the nature of the curve, instead of being irregular as in Fig. 1, is now such that each succeeding positive half-wave is exactly similar to the preceding one, and each succeeding negative half-wave is also exactly similar to those that came before. In fact, the current we are now considering is a *periodic function* of the time: the current rises and falls through a certain cycle of values, which is repeated over and over again;

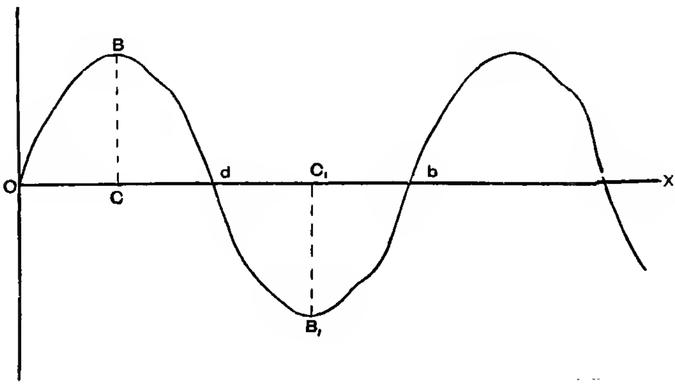

FIG. 2.

and the *periodic time*, or time required for the performance of one complete cycle, is represented in Fig. 2 by the horizontal distance O *b*. This distance, as already mentioned, represents—in the case of the currents we are at present concerned with—only a small fraction of a second of time; and what is known as the *periodicity* of an alternating current is the number of complete cycles performed in one second.

4. **Frequency.**—The word *frequency*, when used in connection with alternating currents, is synonymous with

periodicity, and either word denotes the number of periods or complete cycles per second.

Thus, in Fig. 2, if the distance O b is equivalent to $1 \div f$ seconds, the periodicity or frequency of the current represented in the diagram will be f, and the number of *reversals* or *alternations* per second will be $2f$. In practice the number of periods per second lies usually between 25 and 100. In the early days of alternating currents, when these were used solely for lighting purposes, the periodicity was commonly about 100 in this country; but there are reasons which render a lower frequency advisable, and from 80 to 25 is the usual practice nowadays. In America the standard frequencies are 60 and 25.

5. **Mean and $\sqrt{\text{Mean Square}}$ Values of Alternating Currents.** — Referring again to Fig. 2, it will be readily understood that this curve may stand for any alternating quantity, such as the E.M.F. of a generator, or the current in the filament of an incandescent lamp; and, in any case, whether the curve represents the periodic variation of amperes or volts, the length of the ordinate C B is a measure of the *maximum* positive value of the alternating quantity, and the distance C_1B_1 is a measure of the maximum negative value.

In practice the negative half-wave is generally similar in shape, and equal in magnitude, to the positive half-wave, and it therefore follows that the ordinates C B and C_1B_1 are equal in length. With this *maximum* value of an alternating quantity we shall not concern ourselves at present; it will suffice to point out that this value of an alternating *current* will determine the total magnetic flux or number of magnetic lines linked through the circuit at each reversal; and the insulation of the circuit must be considered in relation to the maximum value of the *voltage* curve.

With respect to the *mean* or average value of an alternating current, it might be supposed that this is the quantity with which we shall be principally concerned; but, as a matter of fact, this is not the case. In the design of alternators, transformers, electromagnetic measuring instruments, etc., it is undoubtedly this value of the induced E.M.F. which is most easily calculated; but without a knowledge of the *shape* of the wave, as shown in Fig. 2, we shall still be without some most important information. It should, perhaps, be hardly necessary to explain what is meant by the *mean* value of an alternating current or E.M.F.: we have merely to take the average of all the ordinates of the wave diagram, or—what amounts to the same thing—measure the area of the curve O B *d* (Fig. 2) and divide by the length O *d*. If a planimeter is not available, recourse can be had to the method frequently adopted in connection with steam-engine diagrams—that is to say, the half-period O *d* would be divided into a convenient number of equal parts, and the sum of the lengths of all ordinates erected on the central point of every section, when divided by the total number of such ordinates, will approximate to the true *mean* value of the current or E.M.F., as the case may be.

Referring still to Fig. 2—which we shall assume represents the variation in strength and direction of an alternating *current* of electricity—let us suppose that the resistance of the circuit conveying the current is R ohms; now, if I is the value of the current at any instant, the rate at which work is being done in *heating* the conductors will, at that particular instant, be equal to $I^2 R$. Hence, if we wish to know the *average* rate at which work is being done by an alternating current, we must calculate the mean value of the *square* of the current and multiply this quantity by the resistance R. Thus, instead of taking

the mean of a large number of ordinates of the half-wave O B d—as in the previous example—we must now take the average of the *squares* of all such ordinates, and this quantity, multiplied by the ohmic resistance of the circuit, will give us the watts lost in heating the conductors.

It follows, therefore, that when we speak of an alternating current as being equal to a certain number of amperes, we invariably allude to that value of the current which, when squared and multiplied by the resistance of the circuit in which it is flowing, will give us the actual power in watts which is being spent in overcoming the resistance of the conductors.

Thus it is the *root-of-the-mean-square* value of an alternating current which, as far as *power* measurements are concerned, enables us directly to compare a periodically varying current with a continuous current of constant strength: it is the product of this value of the current and the corresponding value of the resultant E.M.F. to which it owes its existence which, in all cases, is a measure of the power absorbed in the circuit.

The readings of nearly all commercial measuring instruments for alternating currents depend upon the $\sqrt{\text{mean square}}$ value of the current or E.M.F.; and it is only in exceptional cases that we require to know either the *maximum* or the true *mean* value of an alternating quantity.

If we apply a potential difference of 100 volts to the terminals of an incandescent lamp—the inductance of which is very small, and practically negligible*—the lamp will glow with the same brilliance whether this potential difference is obtained from a continuous or an

* The great importance of the magnetic induction in a circuit conveying an alternating current will be fully dealt with in the following chapter.

alternating current source; and when we speak of an alternating E.M.F. of 1 volt, it is the $\sqrt{\text{mean square}}$ value of the alternating voltage to which we refer. Again, when we speak of an alternating current of, say, 10 amperes, we invariably refer (unless special mention is made to the contrary) to the $\sqrt{\text{mean square}}$ value of the periodic current which, so far as heating effects are

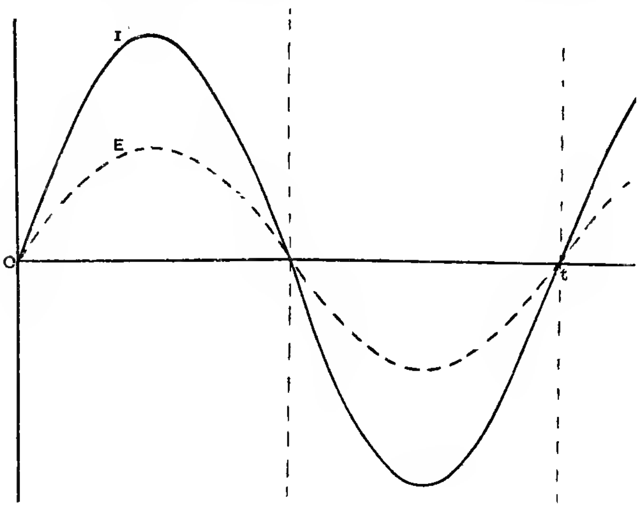

FIG. 3.

concerned, is exactly equivalent to a continuous current of 10 amperes. This value of an alternating quantity is known as the *virtual* or *effective* value.

6. **Phase Difference.**—Two alternating E.M.F.s of the same character and periodicity, or an alternating current and the E.M.F. to which it owes its existence, are said to be *in phase* when the growth and decrease,

reversal, and maximum values (of the same sign) occur simultaneously.

Thus, in Fig. 3, let the full-line curve I represent the periodic variations of the current flowing in a given circuit, and let the dotted curve E represent the alternating E.M.F. in the circuit: the periodicity of these two

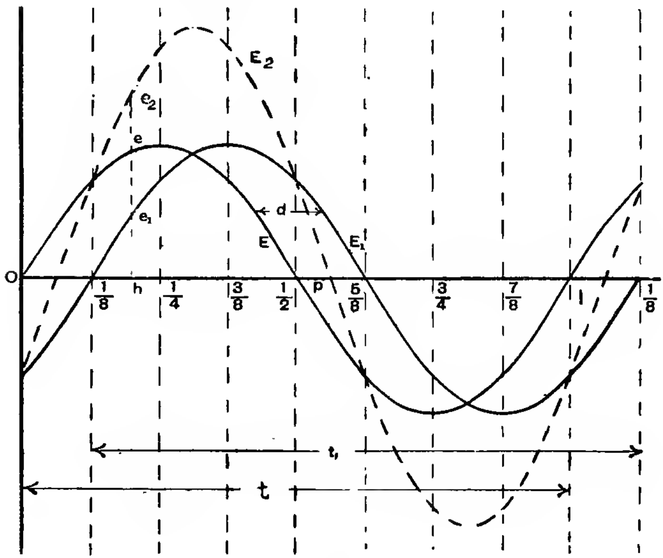

FIG. 4.

quantities is evidently the same, as indicated by the length of the line O *t*—representing the time of one complete period—being equal in both cases. Moreover, it will be observed that the current and E.M.F. pass through zero value, reach their maximum positive value, reverse their direction, and pass through their maximum

negative value at precisely the same instant of time, and they are therefore *in phase.*

Consider, now, the two curves drawn in Fig. 4.

We shall assume that these represent two E.M.F.s of the same wave shape which, for the sake of the argument, might be produced by two alternators, similar in all respects, and having their shafts rigidly coupled together.

These two E.M.F.s are of the same periodicity—the distances t and t_1 being equal—but in this case the two alternating quantities are no longer *in phase.* It will be noted that, throughout the complete cycle, the instantaneous values of the E.M.F. represented by the curve E_1 occur exactly one-eighth of a period after the corresponding values of the curve E ; and E_1 is therefore not in phase with E, but *lags behind* this E.M.F. by a small fraction of a second represented by the distance d, which, in this example, is exactly equal to one-eighth of a complete period. In other words, there is a *difference of phase* between these two E.M.F.s equal to one-eighth of a period.

7. **Sine Waves — Clock Diagram.** — In the last article, the component waves, E and E_1 of Fig. 4, were assumed to be of the same shape, and, as a matter of fact, they are actually *sine waves*, representing a *simple periodic* variation of the E.M.F. This being the form of wave that must necessarily be assumed in all mathematical methods of solving alternating current problems, it is well that the reader should understand what is involved in this assumption.

Consider the so-called *clock diagram* of Fig. 5, in which the length of the line O B is a measure of the *maximum* value of the alternating current or E.M.F. (C B or C_1 B_1 in Fig. 2), and suppose the line O B to revolve

round the point O in the direction indicated by the arrow. If, now, we consider the projection of this revolving line upon any fixed line—such as the vertical diameter M N of the dotted circle—it will be seen that the speed of O B can be so regulated that the length of this projection will, at any moment, be a measure of the instantaneous value of the variable current or E.M.F. It will

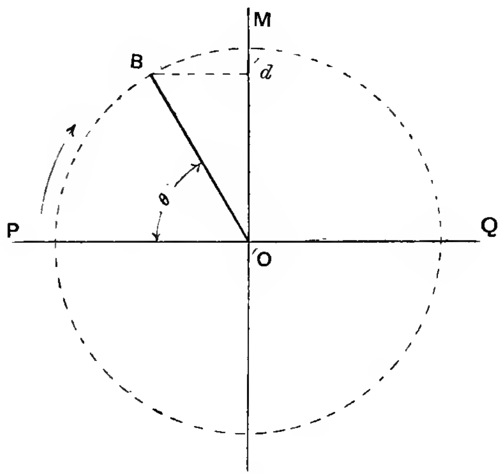

FIG. 5.

also be evident that—since the alternating quantity must pass twice through its maximum value, and twice through zero value, in the time of one complete period—the line O B must, in all cases, perform one complete revolution in $1/f$ seconds; where f is the frequency, or number of periods per second. Also, in order that this diagram may give us all the information needed, it will be convenient to assume that all measurements, such as O d,

which are made *above* the line P Q, refer to *positive* values of the variable quantity, whereas all measurements made *below* will apply to the *negative* values.

When the line O B (Fig. 5) is vertical, its projection O d is equal to it; we therefore conclude that the alternating quantity is at that moment passing through its maximum positive value. As O B continues to move round in a clockwise direction, O d will diminish, until the point B has moved to Q, when O d will be zero; after which it will again increase in length, but this time —since it is now *below* the line P Q—the flow of current or the direction of the E.M.F. is reversed. At N, the maximum *negative* value will be reached, only to fall again to zero at P; after which it rises once more to the *positive* maximum at M.

If the line O B revolves round O at a *uniform* rate, the point d will move to and from the centre O with a simple periodic or simple harmonic motion. It follows that, if the length of the projection O d represents the variations of an alternating E.M.F., this E.M.F. must be understood to be rising and falling in a simple periodic manner; and since the length O d will now be proportional to the *sine* of the *time angle* θ, the shape of the wave (Fig. 4) will be that of a curve of sines, the characteristic feature of which is that every ordinate such as $h\,e$ will be proportional to the sine of its horizontal distance from O; this distance being now measured, not in *time*, but in angular measure, it being understood that 360 degrees correspond to the time of one complete period.

The expression "phase difference" or "angle of lag" has no definite meaning when applied to waves of irregular shape: the lapse of time between successive *maximum* values may not be the same as the interval between successive *zero* values of the quantities compared;

and in the treatment of alternating-current problems, it is therefore usual to assume that the wave shapes are either actually sine curves, or that so-called "equivalent sine waves" have been substituted for the actual waves.

8. **Addition of Alternating E.M.F.s.**—Let us suppose that the two alternators, with shafts rigidly coupled together, generating two alternating E.M.F.s with a phase difference of one-eighth of a period, are electrically joined in series on the same circuit; and let us consider what will be the resultant E.M.F. in the circuit.

When dealing with continuous currents, it is merely necessary to add together the various E.M.F.s—such as those due to a dynamo in series with a battery—paying due regard to their respective *signs*—*i.e.*, whether they are acting in a positive or a negative direction—in order to obtain the resultant E.M.F. producing, or tending to produce, a flow of current in the circuit. And so also in the case of alternating currents, *at any particular instant of time* the resultant E.M.F. in any circuit is equal to the algebraical sum of the various E.M.F.s in the circuit. Thus, on referring again to Fig. 4, it will be noticed that a dotted curve E_2 has been drawn in addition to the two curves E and E_1. Every ordinate of this curve, such as $h\,e_2$, is equal to the sum of the ordinates, $h\,e$ and $h\,e_1$ of the two full-line curves; due attention being paid to the *sign* of these instantaneous values of the E.M.F.

For instance, when the positive value of E_1 is exactly equal to the negative value of E, the resultant E.M.F. in the circuit will be *nil*, and this is what occurs at the instant p where the resultant curve, E_2, passes through its zero value.

A cursory examination of this diagram (Fig. 4) will make it clear that the resultant E.M.F., E_2, is of the same periodicity as the two component E.M.F.s; and

not only its maximum value, but also its virtual value, will be something less than would be obtained by merely adding together the corresponding values of the E.M.F.s E and E_1.*

9. **Vector Diagrams.**—The method described above for graphically adding together two (or more) periodically alternating quantities is very tedious and altogether unpractical. The method about to be described—which involves the proper understanding of a *vector* quantity—is of such general utility in the solution of alternating-current problems that the reader who may not be familiar with these diagrams should devote his most careful attention to the following explanations:

In the first place, a *vector* quantity (as distinguished from a *scalar* quantity) possesses not only *magnitude*, but also *direction.*

A *magnitude* can be graphically represented by the *length* of a straight line, and a *direction* can be represented by the *angle* which a line (of indefinite length) makes with a datum line drawn on the same plan for the purpose of reference; but a vector quantity can only be graphically represented by a line of definite length drawn in a definite direction relatively to some datum line.

The resultant magnetic force at any particular point in space is a vector quantity, because this force has not only a definite direction, but it has also a definite intensity; and if it were customary to indicate, on geographical maps, the magnetic condition at any given place, a line would have to be drawn not only in a certain direction, but also

* It is only when the two component waves are similar in shape, and *in phase*, that the resultant E.M.F. will be exactly equal—both in regard to its maximum and its $\sqrt{\text{mean square}}$ values—to the simple addition of the corresponding values of the component waves.

of a definite length such as to indicate, let us say, the intensity of the horizontal component of the earth's magnetism at that place. Such a line, drawn to scale, with an arrow-head at one end (to indicate direction) would be what is commonly known as a *vector*.

Instead of representing an alternating current or E.M.F. by means of the wave diagrams with which we have been studying them up to the present, let us assume that we are in no wise concerned with the maximum or mean values, or the law of variation of an alternating quantity, but only with its $\sqrt{\text{mean square}}$ or *virtual* value, which, as explained above, is what we most frequently wish to know. This is also the quantity which is measured by an alternating-current ammeter or voltmeter. A straight line of such a length as to indicate the amount or intensity of this alternating quantity drawn *in any direction* on a sheet of paper would be a correct graphical representation of a definite number of amperes or volts. But when dealing with two or more alternating forces all acting in the same circuit, it is necessary that the straight lines representing these forces should be not only of definite lengths; they must also be drawn at certain definite angles relatively to each other in order to represent the phase differences between them.

In Fig. 6 the complete circle of 360 degrees (or 2π radians) represents one period, and in order to indicate the phase difference between the alternating quantities E and I, two straight lines OE and O I—starting from the common point O, and of such lengths as to indicate the respective *magnitudes* of these quantities—are drawn with an angle θ between them such that the ratio which this angle bears to the complete circle expresses the *phase difference* between E and I as a fraction of the complete period.

For instance, if the angle θ is equal to 45 degrees, or $\frac{\pi}{4}$ radians (as in Fig. 6), this means that the current I *lags behind* the E.M.F. E by a time interval equal to one-eighth of a period; or, in other words, the E.M.F. E is *in advance* of the current I by this same interval of time.

Thus, if the *periodicity* of this particular E.M.F. is 40,

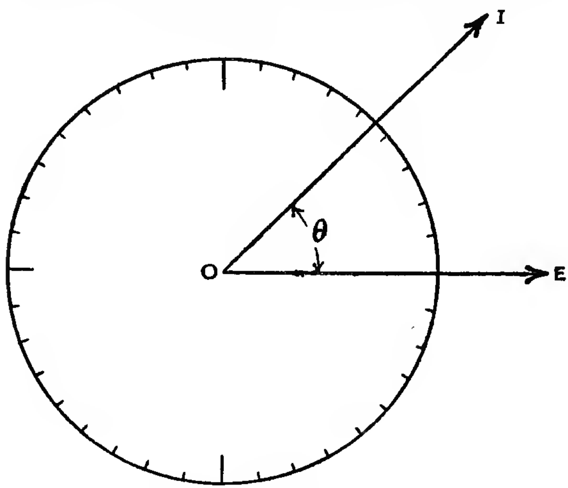

FIG. 6.

it follows that the angle I O E in Fig. 6 represents a time interval of $\frac{1}{320}$th of a second.

It is not usual to express a phase difference as a definite interval of time, because this would convey no useful information, unless the periodicity were also stated. What we wish to know is the fraction of a complete period by which one of the alternating quantities is in advance of the other, and this, as explained above, can

readily be stated as an angle; it is only necessary to bear in mind that the complete circle (360 degrees) stands for one period or double alternation.

If the reader has carefully followed what has been said regarding the graphic representation of alternating quantities by means of *vectors*, it is more than probable that he may yet experience considerable difficulty in forming a clear conception of the phase difference—not between the respective *maximum* or *zero* values, or any other definite instantaneous value of two alternating quantities, but between their *average* or, what is more important, their $\sqrt{\textit{mean square}}$ values.

This difficulty is considered in the following article.

10. **Addition of Alternating E.M.F.s with the Aid of Vector Diagrams.**—Consider once again (as in articles 6 and 8) two alternators, A and B, joined in series: and assume that they are similar in all respects, with the same number of poles, and are driven at the same speed.

Let us suppose three voltmeters to be connected as shown in Fig. 7. These voltmeters must be such as may be used indifferently on alternating or direct current circuits; that is to say, they must measure the $\sqrt{\text{mean square}}$ values of the alternating volts.

The voltmeters E_1 and E_2 will indicate the volts due respectively to the alternators A and B, whereas E will measure the resultant volts at terminals. If the volts measured by E are equal to the arithmetic sum of the volts E_1 and E_2, the two machines would be said to be *in phase;* but, as a rule, the reading on E will be *smaller* than the arithmetic sum of E_1 and E_2. We will suppose these three values to be known. From the centre O (Fig. 8) describe a circle of radius O E, the length of

which is a measure of the volts E. Now draw O E_1 in any direction to represent the volts E_1. From E_1 as a centre describe an arc of radius E_1 E, the length of which is proportional to the volts E_2: it will cut the arc already drawn at the point E. Join O E, and complete the parallelogram O E_1 E E_2. The angle θ between the two component vectors O E_1 and O E_2 is a measure of what we must now understand as the *angle of lag* or *phase difference* between two alternating quantities which are of the same frequency and wave shape.

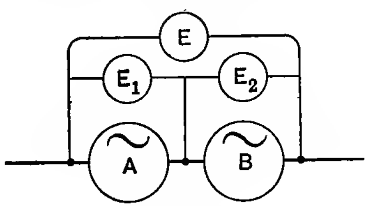

FIG. 7.

The question of compounding two or more alternating forces in an electric circuit now becomes a very simple matter. Thus, in Fig. 8, had we been given the two voltages E_1 and E_2 and the phase difference θ (instead of the three voltages), we could have calculated the total E.M.F., E, and ascertained its phase relation to the two component forces, by merely constructing the *triangle of forces* O E_1 E in the manner familiar to every engineer.

As an example, let us suppose that there are three distinct alternating E.M.F.s—A, B, and C—of the following values, all combining to produce one resultant E.M.F. in an electric circuit:

A = 200 volts.
B = 150 volts.
C = 100 volts.

We shall also assume that B lags behind A by exactly a quarter of a period (90 degrees), while C leads, or is in advance of A, by the fraction of a period denoted by an angle of 35 degrees.

Draw the three vectors O A, O B, and O C, in Fig. 9, to a suitable scale, and in such directions that the angles A O B and A O C are respectively equal to 90 degrees and 35 degrees, bearing in mind that O B must be drawn

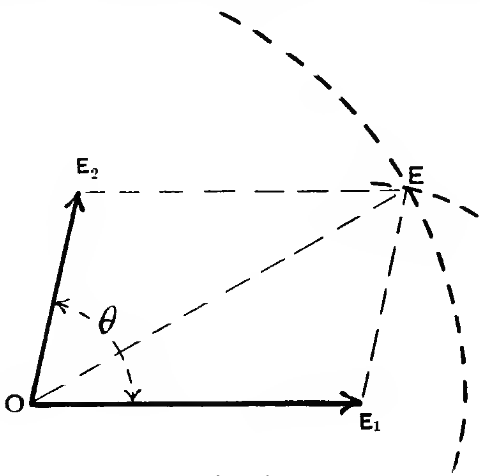

FIG. 8.

behind O A, while O C must be drawn *in advance.** Construct the *polygon of forces* O B *r* R closed by the *resultant* OR, which represents the sum of the three forces—or vectors—O A, O B, O C, and which *lags behind* the vector

* In accordance with the convention adopted by international agreement, the *counter-clockwise* direction indicates *advance in phase.* In this respect the vector diagrams differ from those in the previous edition of this book.

O A by an angle θ equal, in this example, to 18 degrees. The length of the line O R, measured to the same scale as the three component vectors, gives us the value (in volts) of the resultant E.M.F. In this example it will be found to be 297 volts.

Although the diagram Fig. 9 has been drawn to scale, and the problem solved by taking actual measurements

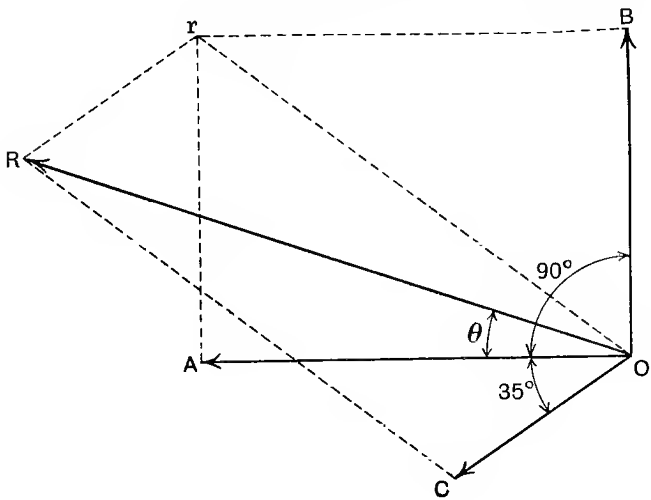

Fig. 9.

off the diagram, it should be stated that this method will usually be found unpractical, mainly because of great differences in the magnitudes of the quantities dealt with, and the difficulty experienced in locating the exact point of intersection of two lines when the angle between them is small. It will generally be found convenient to draw the vector diagram (not necessarily to scale), and then

solve for the unknown quantities by using trigonometrical formulas or vector algebra. When using the algebraic method, every vector is resolved into its vertical and horizontal components, and the various operations are performed on these components only. The addition of several alternating currents of the same frequency therefore resolves itself into the process of finding the resultant of several vectors, much as a surveyor would compute the result of a day's traverse by "latitude and departure."

11. **Mean Power of an Alternating Current.**—In article 8 (p. 13), the two full-line curves in Fig. 4 were supposed to represent two E.M.F.s, and it was shown how these could be *added together* to produce the resultant curve E_2. Let us now consider, not two E.M.F.s differing in phase, but an E.M.F. and a current, also differing in phase; and let us *multiply* one by the other in order to obtain the *power* in the circuit.

From what has been said on the subject of phase differences generally, the reader should have no difficulty in understanding that, whereas in the case of continuous currents the power in a circuit is given by the product of terminal potential difference and total current, this is very rarely true in the case of alternating currents.

If we connect a voltmeter across the terminals of an alternating-current circuit, and multiply the reading on this voltmeter by the actual (virtual) value of the current in amperes, we shall obtain a number representing what is sometimes called the *apparent power;* but this will not necessarily be a measure of the power actually being supplied to the circuit. The true power supplied to the circuit at any moment will be given by the product of the instantaneous values of current and impressed potential difference, and the mean value of all such products, taken

during the time of one complete period, will be the quantity which we require to know.

In Fig. 10 the power—or watt—curve has been drawn; it is obtained by multiplying together the corresponding ordinates of the curves of impressed potential difference E and of the current I.

Since the current lags behind the potential difference, it follows that, during certain portions of the complete

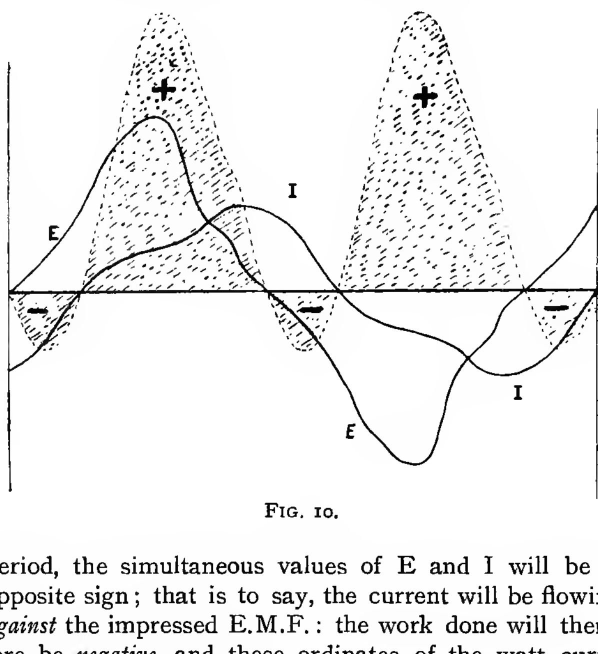

FIG. 10.

period, the simultaneous values of E and I will be of opposite sign; that is to say, the current will be flowing *against* the impressed E.M.F.: the work done will therefore be *negative*, and these ordinates of the watt curve will have to be plotted *below* the datum line. This negative work (which is equal to the area of the shaded curve below the datum line) may sometimes almost equal

the *positive* amount of work done, in which case the current is practically *wattless—i.e.*, the amount of energy put into the circuit during one quarter period is given back again during the succeeding quarter period.

(The proper understanding of this state of things must of necessity present some difficulties to those unacquainted with alternating currents; but when dealing with the reactance of an alternating-current circuit, the matter will be again referred to.)

Returning to a consideration of the curves of Fig. 10, we see that the total amount of work done during one complete period is equal to the area of the two shaded curved curves marked +, *less* the area of the two shaded curves marked –; and the average *power* supplied to the terminals of the circuit will, therefore, be indicated by the average ordinate of this dotted *watt* curve, due attention being paid to the *sign* of the instantaneous power values.

Instead of drawing a diagram such as Fig. 10, which is a very laborious undertaking, let us see what can be done with the simpler and more convenient vector diagrams.

In Fig. 11 the $\sqrt{\text{mean square}}$ value of the E.M.F. is represented by the vector O E, and the $\sqrt{\text{mean square}}$ value of the current by O I.

The phase difference between these two alternating quantities is indicated by the angle θ; and since O E is in advance of O I (refer to footnote on p. 19), it follows that the current I *lags behind* the E.M.F., E.

The method of compounding or adding together two or more E.M.F.s was explained in article 10 (p. 17); and it follows from this that any vector, such as O E, representing an E.M.F., can be considered as being made up of two or more component vectors.

Thus, in Fig. 11, if we draw through O the line O B at right angles to O I, and from the point E drop perpendiculars E e and E e_1 on to these two lines, we have in O e and O e_1 two vectors representing imaginary E.M.F.s which, when added together in the manner explained in article 10, would produce the resultant E.M.F., E.

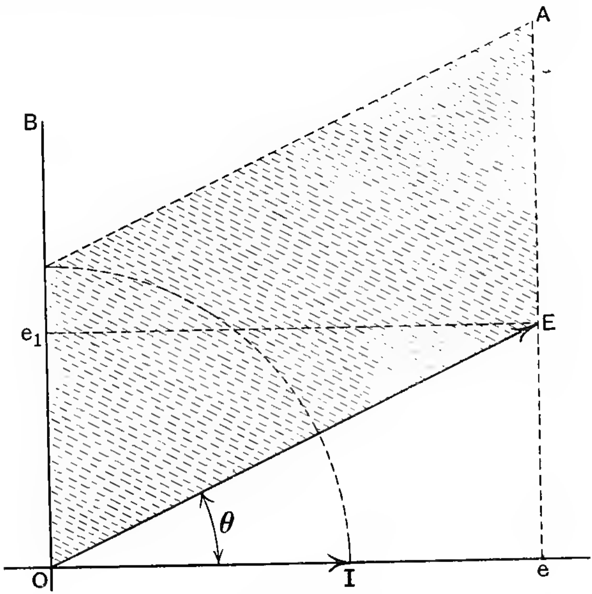

FIG. 11.

Suppose, now, that there is no other E.M.F. in the circuit but that represented by the vector O e. This is in phase with the current, and the true watts are therefore obtained by multiplying together this E.M.F., e, and the current I.*

* See article 5, especially the concluding paragraphs (p. 7).

If, on the other hand, e_1 were the only E.M.F. in the circuit, the current I would be *wattless—i.e.*, the mean power in the circuit would be *nil*—because, when the phase difference between the current and E.M.F. is 90 degrees, the energy put into the circuit during one quarter of a period is given back again during the next quarter of a period, and the mean watts are, therefore, equal to zero.

This leads us to the conclusion that, of the two (imaginary) components, e and e_1, of the total E.M.F., it is only the component e, *in phase with the current*, which need be taken into account when calculating the power in the circuit; and, given the angle of lag (or phase difference) between the applied potential difference E and current I, the total power in the circuit is obtained by projecting one of the vectors—such as O E—upon the other—O I—and then multiplying this latter quantity by the projection (O e) of the first one upon it.

In order to obtain a graphical representation of the power supplied to a circuit in which the current I lags behind the impressed volts E by an amount equal to the angle θ, we have simply to move round one of the vectors, let us say O I, through an angle of 90 degrees, and then construct the parallelogram O A, the area of which will be a measure of the average value of the true watts; for it is evident that this area will always be equal to O I $\times$ O e.

12. **Power Factor.**—Since e O E (Fig. 11) is a right-angled triangle, it follows that the power (I $\times e$) supplied to the circuit can be written I $\times$ E $\cos\theta$. But I $\times$ E is what we have already called the *apparent watts*. Hence the *real watts* = the *apparent watts* $\times \cos\theta$, and this is the definition of the *angle of lag* between current and impressed E.M.F., which is the most useful. It is this multiplier ($\cos\theta$) to which the name *power factor* has been

given. The power factor of a circuit-carrying current under definite conditions is the ratio of the true power to the apparent power or *volt-amperes.* It cannot be greater than unity, and, in a circuit carrying an alternating current, is usually less than unity.*

As this expression occurs frequently in all literature dealing with alternating currents, it is important that the reader should have a clear understanding of its meaning.

In connection with continuous currents, the term is not used, for the simple reason that, once a steady flow of current has been established, the value of this current is always equal to the ratio of *applied E.M.F.* to ohmic resistance of the circuit; and the loss of power in the circuit may either be written $I^2 R$ or E I.

In the case of alternating currents, it very often happens that the current is not *in phase* with the applied E.M.F., and the causes leading to this displacement of phase are sufficiently important to justify our devoting the following chapter to their consideration. The effects of such phase displacement between current and E.M.F. have, however, already been explained, and it will be understood that, although the power spent in heating the conductors of the circuit may still be written $I^2 R$, the product $E \times I$ only represents the true power when the *power factor* is

* It has been assumed that the reader is not without at least an elementary knowledge of trigonometry; but should this assumption be unwarranted, it will suffice to point out that the *cosine* of the angle θ is the trigonometrical function of the angle, the numerical value of which is always obtained by dividing the length of the side O *e* of the right-angled triangle E O *e* by the length of the hypotenuse, O E. The expression $\cos \theta$ must, therefore, be understood to indicate the ratio of O *e* to O E, which ratio will, obviously, be constant for a given angle, and quite independent of the actual lengths O E and O *e*.

unity—*i.e.*, when the current is in phase with the applied E.M.F. Whenever there is the slightest phase displacement between the applied E.M.F. and the resulting current, the *apparent power* (E × I) has to be multiplied by the *power factor* (*cos* θ) in order to arrive at the true power in the circuit.

CHAPTER II

SELF-INDUCTION AND CAPACITY

13. **Magnetic Field Due to a Continuous Current.**—Wherever there is a current of electricity there must, of necessity, be a corresponding magnetic condition of the surrounding medium.

In the case of continuous currents, once a steady value has been reached, the magnetic condition—*i.e.*, the number and direction of the magnetic lines in the circuit—remains unaltered. Thus, so long as the resultant exciting ampere-turns in a dynamo machine or direct-current motor remain constant, the total magnetic flux is maintained at a definite steady value. In order to calculate this total flux, it is necessary to know, not only the current and the number of turns in the exciting coils, but also the dimensions and configuration of the magnetic circuit which is linked with the electric circuit, and especially the amount and disposition of any masses of iron in the direct path of the magnetic lines.

It is assumed that the reader has a fair working knowledge of the more important properties of the magnetic circuit; he should understand, for instance, the manner in which the total magnetic flux in the armature of a dynamo can be approximately predetermined, for without a clear conception of the fundamental laws of the magnetic circuit, including the peculiar property of iron

(and one or two other metals) of *increasing* the magnetic flux, the following arguments may not be readily understood.

Apart from the $I^2 R$ losses in conductors, no work has to be done in order to *maintain* a magnetic field; but energy was spent in *creating* it, and this energy will all be given back again to the exciting circuit when the magnetic field is annulled or withdrawn.

In order to form a mental picture of this property of an electric circuit, consider a flywheel, and neglect entirely all questions of bearing friction or windage—which, in our analogy, are equivalent to the $I^2 R$ losses referred to above. Such a flywheel, once it has attained a definite speed of rotation, will continue to revolve for any length of time without requiring the further application of force. But a force had to be applied to bring it up to speed, and exactly the same amount of energy as was put into it is now available for doing work, and will be given back again by the time the flywheel has been brought to rest.

If we multiply the total flux in the core of a dynamo field-magnet by the number of turns in the exciting coil, we obtain a quantity which is sometimes referred to as the *electromagnetic momentum* of the circuit. There is energy stored up in such a circuit, and if we attempt to open it suddenly, the results are frequently disastrous, because this energy *must* be transferred or dissipated in one form or another, and if we do not provide a short-circuiting resistance, or some means of drawing out the arc slowly, the dying down of the magnetism will induce such a high E.M.F. in the field coils as to break down the insulation or form a destructive arc at the point of disconnection. If, on the other hand, we short-circuit the winding, or connect its terminals—at the moment of switching off the supply—through an external

resistance, the current in the coil, instead of being almost instantly interrupted, will die down in the manner shown on the diagram Fig. 12.

Here the horizontal distances from left to right represent lapse of time, and the vertical distances represent current. It will be noted that the current falls rapidly at first, but its rate of decrease diminishes as time goes on. Theoretically, the current never quite reaches zero

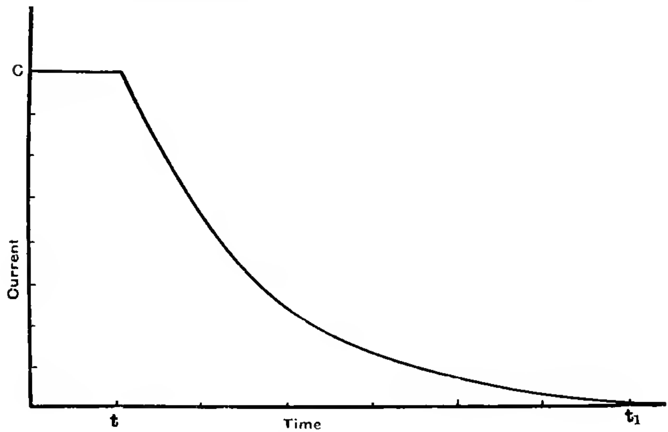

FIG. 12.

value; but, in practice, it approximates to zero value in a very short space of time. In most cases, the whole distance from t to t_1 would stand for only a small fraction of a second; but in the case of a circuit with large *self-induction* such as the field coil of a dynamo, this time may extend over several seconds. The writer recollects receiving a shock from a large direct-driven two-pole dynamo, *after the same had been slowed down and come to a standstill*, at the moment of lifting the brushes off the

commutator. This was entirely due to the current in the field coils—which were short-circuited through the armature—not having quite died down, and when the circuit was opened, the energy still stored in it took the form of a small current at a high potential. It should not be necessary to point out that, when the current dies down in the manner indicated in Fig. 12, the total energy put into the circuit when building up the field is given back in the form of $I^2 R$ losses continuing for an appreciable *time* in the coil itself, and the external resistance (if any) which is connected across the terminals.

The idea of the magnetic condition always existing simultaneously with the electric current must not be lost sight of. Thus, in Fig. 12, the reason why the current does not drop instantly to zero is briefly this: if it were to do so, the magnetic flux in the core (apart from the *residual* magnetism) would of necessity have to drop likewise instantaneously; but as, in so doing, it would induce a back E.M.F. of infinite value, and such as would tend to produce a current opposing the withdrawal of the magnetism; it follows that the current (and magnetism) in a circuit of appreciable resistance will actually die down in the manner indicated in Fig. 12; every decrease in the current (and magnetism) producing a certain *E.M.F. of self-induction* in the coil, tending to oppose a more rapid rate of decrease.

14. **Magnetic Field Due to Alternating Current.**—When an alternating current flows in a circuit, the magnetic condition of the surrounding medium will vary in amount and direction in accordance with the variations of the current. For any given circuit, the amount of the magnetic flux may be approximately predetermined for any instantaneous value of the current, provided we know the length and cross section, or the *magnetic resistance* of

the various parts of the magnetic circuit, and the number of turns in the electric circuit.* If there is no iron or other *magnetic* material in the magnetic circuit, the rise and fall of the magnetism will synchronise with the rise

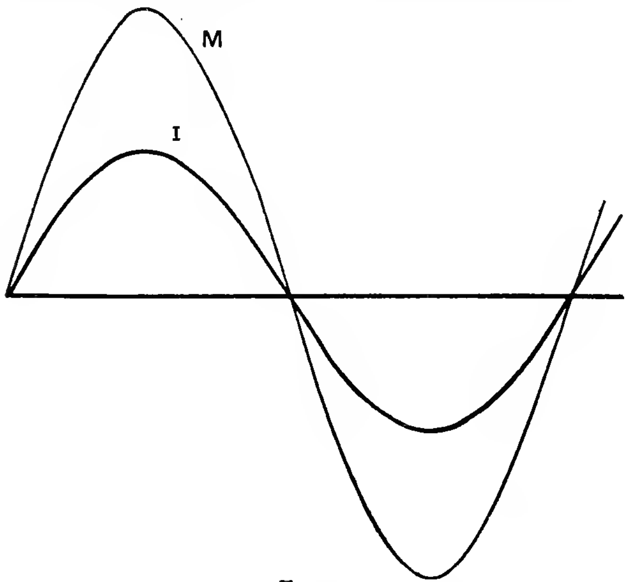

FIG. 13.

and fall of the current, and not only its maximum value, but also every intermediate value will be exactly propor-

* If there is iron in the magnetic circuit, the effects of *hysteresis* must be taken into account. It should not be necessary to remind the reader that, when carrying a piece of iron through a complete cycle of magnetisation, the ampere-turns corresponding to a definite value of the induction on the rising portion of the cycle are not equal to those corresponding to the same value of the induction on the descending portion—*i.e.*, when the current, after having passed through its maximum value, is being reduced.

tional to the strength of the current: when the current reaches zero value, the magnetism will also pass through zero value; and when the current reverses in direction, the magnetic flux will reverse likewise.

Thus, if we plot the magnetism corresponding to every value of the current in a given circuit *containing no iron*

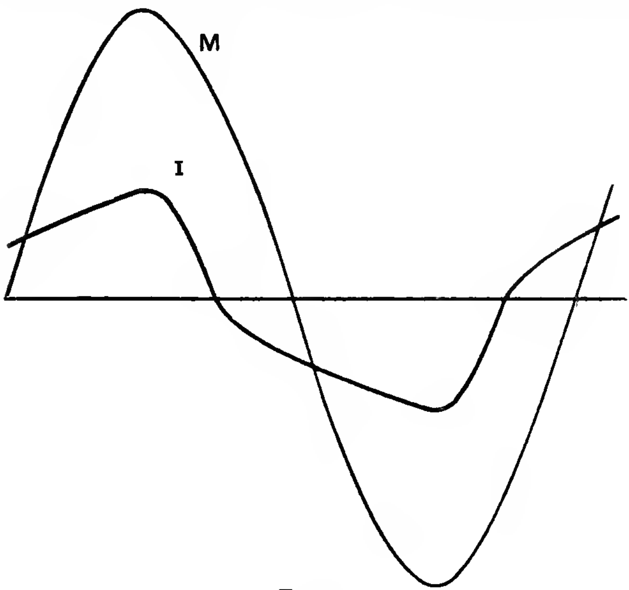

FIG. 14.

(or other magnetic metal), we obtain a curve such as that shown in Fig. 13, where I is the current wave and M the magnetism. Every ordinate of this latter curve is equal to the corresponding ordinate of the curve I multiplied by a constant, and the magnetism is, of course, exactly *in phase* with the current.

In Fig. 14 the curve M is such as would be obtained

if the magnetic circuit contained a large amount of iron. Here, with the one exception that the maximum value of the induction coincides with the maximum value of the current, it will be noticed that, on the whole, the magnetism *lags behind* the current.*

A full discussion of the various effects of *hysteresis*, or even a detailed description of the manner in which the curve M in Fig. 14 is derived from the curve I, hardly enters into the scope of the present book; but the reader requires only an elementary knowledge of magnetic phenomena—especially as regards the effects of iron in a magnetic field—to understand that the *residual magnetism*, which requires a reversed magnetising force to eliminate it, is responsible for the peculiar relation of the two curves under consideration.

15. **E.M.F. produced by an Alternating Magnetic Field.**—We know that the E.M.F. generated in a conductor passing through a magnetic field is directly proportional to the rate at which the conductor is cutting the (imaginary) magnetic lines—or tubes—of induction. If 100,000,000 maxwells or C.G.S. magnetic lines are cut during one second of time, the average value of the resulting E.M.F. will be 1 volt. If we thrust a magnet into a coil of wire, the E.M.F. generated in the coil is proportional to the strength of the magnet and the rapidity with which it has been thrust into the coil. The *direction* of this E.M.F. is always such as to tend to produce a current which will *oppose* the alteration in the magnetic condition.† If, on inserting the magnet, a positive E.M.F.

* Hence the word *hysteresis* originally suggested by Professor Ewing.

† There is no exception to this rule, which is known as Lenz's law.

was generated, a negative E.M.F. will be induced when the magnet is withdrawn.

In Fig. 15, let the curve M represent the rise and fall of the alternating magnetism due to an alternating electric current—which has not been drawn in the diagram, but which, if there is no iron in the circuit, will be in phase with M, and, if there is iron in the circuit, will be *in*

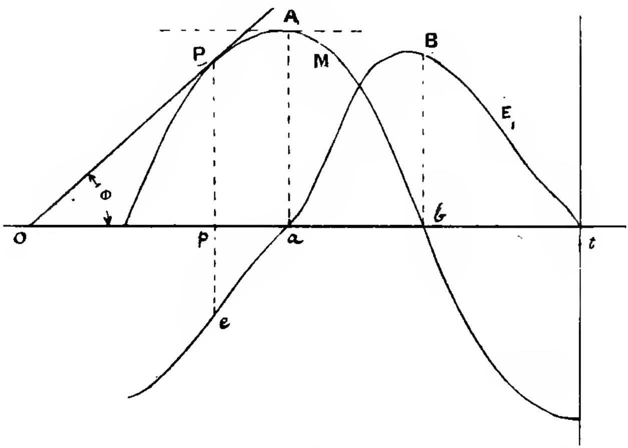

FIG. 15.

advance of M (see Figs. 13 and 14). Let us see what s the E.M.F. which this varying magnetism will induce in an electric circuit with which it is linked. In the first place, it is evident that, if the magnetism were of a constant value—however large—no E.M.F. would be generated in the circuit. In other words, if its *rate of change* were *nil*, the value of the induced E.M.F. would be zero. This is exactly what occurs at the point A, when the curve M has reached its maximum value; the amount of magnetism is neither increasing nor decreasing

at this particular instant, and consequently there can be no E.M.F. generated. We can, therefore, plot the point *a* of the E.M.F. curve on the horizontal datum line immediately below the maximum value of the magnetism curve—this maximum value being graphically defined by the point of contact of the horizontal tangent to the curve M, as indicated by the dotted line.

Consider, now, the point *b* on the curve M. This point represents not only the instant of time corresponding to the reversal in direction of the magnetism; but, in this example, it also indicates that portion of the cycle where the *rate of change* in the magnetism is greatest—as evidenced by the *slope* of the curve being steepest at this point. If this change is at the rate of 100,000,000 C.G.S. magnetic lines withdrawn from the circuit *per second*, an E.M.F. of 1 volt will be generated *in every turn of the electric circuit* which is linked with this magnetism.

Thus, if we know the number of turns in the circuit and also the scale to which the curve M has been drawn, we can readily calculate the actual induced E.M.F. at the instant *b* and plot this value *b* B to any convenient scale.

As to the *sign* of this E.M.F., it will, at this point, be *positive* (and therefore must be plotted *above* the datum line) because, since the magnetism is falling from a positive maximum towards a negative maximum, it is a *positive* E.M.F. which is necessary to produce a current such as would *oppose* or *counteract* this decrease in the magnetism.

We have seen that, where the curve M is horizontal, the induced volts are *nil;* if it were possible for this curve to drop so rapidly as to become perpendicular to the horizontal datum line, this would indicate an *instantaneous* change in the magnetic condition, or an infinitely

great *rate of change*, and, therefore, an infinitely great induced E.M.F. whatever might be the actual number of magnetic lines added to or withdrawn from the circuit; and if the reader has a clear understanding of these extreme conditions he will probably accept the statement that the *steepness* or *slope* of the curve M is, at every point, an exact measure of the rate of change in the magnetic condition.*

As an example, suppose the ordinate p P represents 100,000 C.G.S. lines of magnetism. Draw the tangent O P to the curve M at the point P, and measure O p: let us assume that this distance corresponds to the two-hundredth part of a second. The increase in the magnetism at the point P is, therefore, at the *rate* of 100,000 C.G.S. lines in the two-hundredth part of a second, or twenty million lines per second; and if we assume that there are one hundred turns of wire in the circuit, the instantaneous value of the induced E.M.F. at the moment p in the hundred turns of wire will be

$$\frac{20,000,000 \times 100}{100,000,000} = 20 \text{ volts},$$

and this value, represented by $p\ e$, must be plotted *below* the datum line, because this E.M.F. will be such as will tend to produce a current in a *negative* direction—*i.e.*, such as would *oppose* the variation in the magnetic flux, which, at this moment, is *increasing* in amount.

The circuit we have been considering might be the armature winding of an alternator or the secondary coils

* It would be an easy matter to prove this statement, and in all probability the reader will be quite capable of doing this to his own satisfaction. The author's purpose is not to prove every statement made in the course of this and subsequent chapters; but a general explanation, suggestive of the lines on which more exact proofs may be sought, will, wherever possible, be given.

of a transformer, and, in either case, the induced E.M.F., as shown in Fig. 15, would be a quarter period—or 90 degrees—*in advance* of the alternating magnetism to which it owes its existence; and, moreover, since exactly the same total magnetic flux as is threaded through the coils during one half-period is withdrawn during the succeeding half-period, the *mean* value of the positive half-wave of the induced E.M.F. is always exactly equal to the mean value of the negative half-wave.*

If N is the total amount of magnetic flux produced by the maximum value of the alternating current passing through the coil, and if S stands for the number of turns in the coil and f for the frequency, then the *mean* value of the induced E.M.F. is proportional to N S f, and this relation holds good whatever may be the shape of the current wave producing the magnetism. (See Appendix I. at end of book.)

It must not be supposed that because the *mean* value of the E.M.F. is independent of the wave form, the $\sqrt{\text{mean square}}$ value is likewise unaffected by the shape of the wave: on the contrary, the $\sqrt{\text{mean square}}$ value bears no definite relation to the mean value, but depends largely upon the wave form, which it is very difficult to predetermine with accuracy.

16. **Inductance.**—The changes in the magnetic flux of induction linked with a given electric circuit, as represented by the curve M of Fig. 15 in the preceding article, may conceivably be due to changes in the current carried by this particular circuit. In such case, the curve E_1 would represent the *counter E.M.F. of self-induction.* The self-induction of a circuit, as indeed is indicated by the

* The *time integrals* of the positive and negative half-waves are equal, whatever may be the law of variation of the alternating magnetism.

expression itself, is the induction or total number of magnetic lines *due to the current flowing in the circuit.* For any particular current I in the conductor forming the electric circuit, the total magnetic flux of induction linked with this circuit *which is due to the current I* is the *self-induction* corresponding to that particular current.

What is known as the *coefficient of self-induction*, or, more simply, the *inductance* of a circuit, is a multiplier—generally denoted by the letter L—which takes into account, not only the amount of the total flux of induction due to unit current flowing in the circuit, but also the number of times the induction is linked with the electric circuit. Thus the inductance, L, might be defined as the amount of *self-enclosing* of magnetic lines by the circuit when the current has unit value. It cannot be expressed in *maxwells*, since it is equal to *maxwells* × *number of turns*. The practical coefficient of self-induction is the *henry*. If the number of maxwells representing the flux **N** is known when the maximum value of the current is I amperes, and if the circuit carrying the current I is linked S times with the flux **N**, then the inductance, in henrys, is

$$L = \frac{\mathbf{N}\ S}{10^8 \times I}.$$

This coefficient is constant only for a circuit containing no iron. The presence of iron leads to the flux **N** being no longer proportional to the current I, and L will therefore be a function of I.

It is not proposed to make much use of the quantity L in this book; but it is convenient, and of frequent occurrence, in the analytical treatment of alternating-current problems.

17. **Reactance.**—In a circuit carrying a continuous current, the E.M.F. can be expressed in terms of the

current and resistance by writing Ohm's law in the form $E = I \times R$. We have just seen how there is another E.M.F., called the E.M.F. of self-induction, which very frequently asserts its presence in a circuit carrying an alternating current, and the term *reactance* has, for convenience, been given to the ratio obtained by dividing the E.M.F. of self-induction by the current in the circuit. Thus we may write.

E.M.F. of self-induction = current × reactance,

the last term being a multiplier which can be expressed in ohms, and which will be directly proportional to the inductance of the circuit and to the frequency of the current, but which will also depend, to a certain extent, upon the wave form of the alternating current.

Thus, if a circuit has large inductive reactance, the induced E.M.F., for a given current, will be greater than if the reactance is small. A load of incandescent lamps is an example of a circuit of small reactance, whereas the primary winding of a transformer—which, in addition to enclosing a large magnetic flux, has also a considerable number of turns—is a circuit of large reactance; but this expression has *no meaning* except in relation to a circuit carrying an alternating current at a definite *frequency*. In fact, the chief reason why reference has been made to this term is that it is used by most writers when treating of alternating currents, and without an elementary knowledge of the meaning of such terms, the reader might be seriously handicapped when taking up more advanced works on the subject.

If it is permissible to assume the sine wave variation of E.M.F. and current, it can be shown that the inductive reactance is

$$X = 2\pi f L,$$

where L is the coefficient of self-induction of the circuit, expressed in henrys. The counter E.M.F. of self-induction is, therefore,

$$X I = 2 \pi f L I,$$

where I is the R.M.S. value of the current, in amperes, if it is the corresponding value of the E.M.F. that it is required to calculate.

18. **Current Flow in Circuit of Negligible Inductance.**—Let us consider an electric circuit which is practically without self-induction, or electrostatic capacity. It may consist of a wire doubled back upon itself (in the manner adopted in winding resistance coils for testing purposes), or of glow lamps, or of a water resistance.

If an alternating E.M.F. is applied to the terminals of such a circuit, the current at any instant will be equal to the ratio of the instantaneous value of the E.M.F. to the total resistance of the circuit; or,

$$I_{inst.} = \frac{E_{inst.}}{R},$$

from which we see that the current wave will be of the same shape as the E.M.F. wave, and *in phase* with it—a state of things which is the evident result of the fulfilment of Ohm's law: for there is no reason for supposing that Ohm's law is not equally applicable to variable as to steady currents; it is only necessary to bear in mind that, in the case of variable currents, the *applied* E.M.F. and the *resultant* E.M.F. in the circuit (to which the current is due) are not necessarily one and the same thing. In the case under consideration, of a circuit supposed to be without self-induction or capacity,* there is only one E.M.F. tending to produce a flow of current—*i.e.*, the

* The effects of capacity will be dealt with in due course.

E.M.F. supplied at the terminals of the generator: the current will therefore rise and fall in exact synchronism with the applied E.M.F.

19. **Current Flow in Circuit of Appreciable Inductance.**—In order to get a better understanding of the whole question of self-induction in connection with alternating currents, let us consider an alternating current flowing in a circuit which has both ohmic resistance and reactance—as, for instance, a coil of wire of many turns, which, for the present, we will assume, has no iron core. Such a current is shown graphically by the curve I in Fig. 16, where intervals of time are measured, as usual, horizontally from left to right. The magnetism due to the current I will vary in amount and direction in accordance with the variations of the current. It may be calculated in the usual way for any given value of I, provided we know the length and cross-section, or the *magnetic resistance* of the various parts of the magnetic circuit, and the number of turns of wire in the coil. Let the curve *m* represent the rise and fall of this magnetism.

Since the *induced* or *counter* E.M.F. due to these variations in the magnetic induction will be proportional to the *rate of change* in the total number of magnetic lines threaded through the circuit, we shall have no difficulty in drawing the curve E_1 (as in article 15, p. 34) to represent the E.M.F. of self-induction, which lags exactly one-quarter of a period behind the current wave.

We are now in a position to determine the potential difference which must exist at the terminals of the circuit in question, in order that the current I will flow through it.

Draw the curve E_2 to represent the E.M.F. required to overcome the ohmic resistance. It will be in phase with the current, because its value at any point is simply

$I \times R$, where R stands for the resistance of the circuit. Now add the ordinates of E_2 to those of an imaginary curve exactly similar but opposite to E_1, and the resulting curve E will evidently be that of the impressed potential difference which, if maintained at the ends of the circuit under consideration, will cause the current I to flow in it. Thus we see how the relation between the impressed E.M.F. and the resulting current may be graphically worked out for any given case.

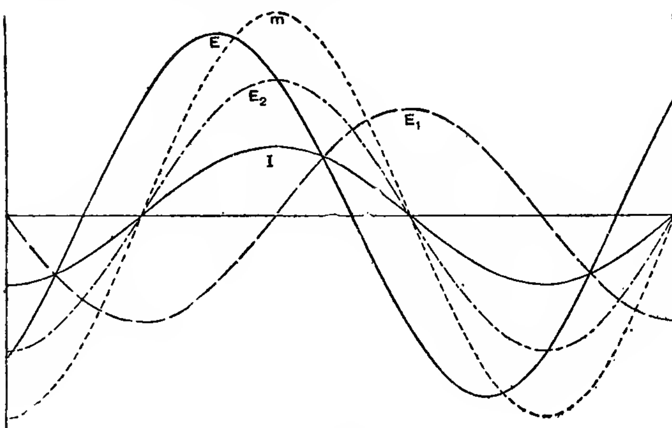

FIG. 16.

From a study of the curves in Fig. 16 it is evident that the effect of self-induction is to make the current lag behind the impressed E.M.F. If the E.M.F. required to force the current against the ohmic resistance is small in comparison with the induced E.M.F., the lag will be very considerable; it cannot, however, exceed one-quarter of a complete period, which limit is only reached when the E.M.F. of self-induction is so large, and the ohmic resistance of the circuit so small, as to render

the E.M.F. required to overcome this resistance of no account.

In order briefly to sum up the principles governing the flow of an alternating current in an inductive circuit, we may say that the varying current produces changes of magnetism, which again produce a varying E.M.F., called the "E.M.F. of self-induction." This, together with the E.M.F. already existing (and without which no current would flow), produces the *useful* or *resultant* E.M.F. By dividing the value of this resultant E.M.F. at any instant by the total ohmic resistance of the circuit, the corresponding current intensity is obtained. This condition *must* always be fulfilled, otherwise Ohm's law would not be satisfied.

20. **Diagram showing Relation of E.M.F.s in Inductive Circuit.**—We have seen in article 15, p. 34, and again in discussing the curves of Fig. 16, that the induced E.M.F. lags exactly 90 degrees behind the magnetism; and since this magnetism rises and falls in synchronism with the current to which it owes its origin, the same relation—*i.e.*, a phase difference of a quarter period—exists between the induced E.M.F. and current in an inductive circuit, such as we have been considering.

Again, the resultant E.M.F. required to overcome the ohmic resistance of the circuit is necessarily in phase with the current (refer to the curve E_2 in Fig. 16), and both these relations hold good, whatever may be the shape of the current wave, provided always that there is no iron or other *magnetic* metal in the circuit. If, therefore, we wish to draw the vector diagram of the E.M.F.s in such an inductive circuit, the two vectors O E_1 and O E_2 (Fig. 17),* representing respectively the

* Compare with Fig. 11, p. 24.

induced and the useful or *energy* E.M.F.s, must be drawn at right angles to each other, with O E_2 in *advance* of O E_1.

The necessary impressed E.M.F. at the terminals of the circuit is found—as explained in article 19—by compounding the E.M.F. E_2 with an imaginary E.M.F. (O *e*) exactly equal, but opposite in phase to E_1. In this manner the vector O E is obtained.

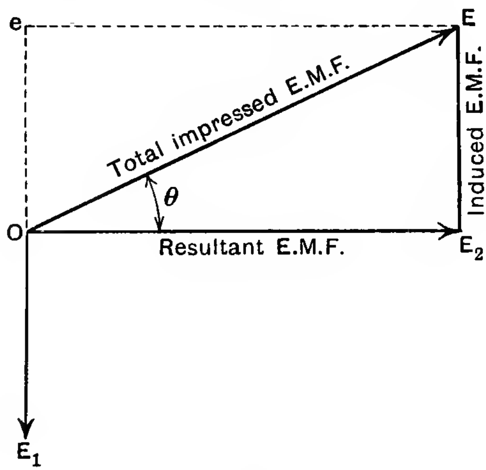

FIG. 17.

Now complete the triangle E O E_2, and note that the lengths of the sides of such a triangle represent respectively the resultant or useful E.M.F.; the counter E.M.F. or E.M.F. of self-induction; and the total impressed E.M.F. at the terminals of the circuit. Moreover, since the side O E is the hypotenuse of a right-angled triangle, this latter quantity (the impressed E.M.F.) is always equal to the square root of the sum of the squares

of the other two, which is an easy and useful rule to bear in mind.

It should not be necessary to remind the reader that the angle θ is the *angle of lag* between the virtual and the impressed volts, and that *cos* θ, or the ratio $O\,E_2 \div O\,E$ is the *power factor* of this particular circuit.

In calculating the various E.M.F.'s in the circuit, as drawn in Fig. 17, the value of the current has necessarily been taken into account. Let us eliminate this factor entirely, by dividing the three quantities $O\,E_1$, $O\,E_2$, and $E\,E_2$ by the value of the current. The

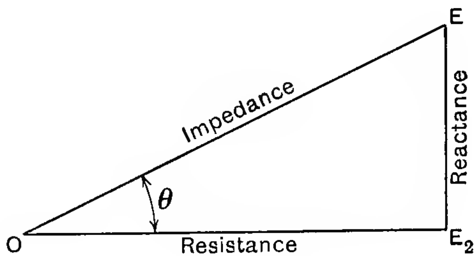

FIG. 18.

triangle need not be altered in any way; it has merely to be considered as having been drawn to a different scale. It is reproduced in Fig. 18, and it will be noted that the line $O\,E_2$ now stands for the *resistance* of the circuit, while $E\,E_2$ represents the *reactance* (see article 17, p. 39). With regard to $O\,E$, since this is the result obtained by dividing the total E.M.F. by the current, it evidently stands for the *apparent resistance* of the circuit, to which the name *impedance* has been given: this quantity, and also the reactance, can be expressed in ohms.

We may, therefore, write:

$$\textit{impedance} = \frac{\textit{impressed volts}}{\textit{amperes}};$$

and again,

$$\textit{impedance} = \sqrt{(\textit{resistance})^2 + (\textit{reactance})^2}.$$

It must not be overlooked that such expressions and relations as these have only a limited usefulness, and the quantity understood by the word *impedance* is merely a multiplier, which is not constant, even for a given circuit, but which depends upon the frequency and wave form of the impressed E.M.F.

A little consideration will make it clear that the higher the periodicity, the more important becomes the *reactance* relatively to the *resistance*, and that, in all cases where the counter E.M.F. is objectionable, it is advisable to keep the periodicity as low as possible.

21. **Effect of Iron in Magnetic Circuit.**—Although it is not intended to investigate the causes which lead to a distortion of the current wave, and a certain loss of energy when iron is introduced into the magnetic circuit, it will, nevertheless, be necessary to note briefly the effects produced when iron is used—as in generators, motors, and transformers—for the purpose of increasing the magnetic induction and concentrating it at certain points.

There are two causes of loss of energy when the path of the magnetic lines is through iron; the first is due to eddy currents, and the second to hysteresis.

22. **Eddy Currents.**—Whenever a conductor of electricity is placed in a fluctuating magnetic field, Foucault or eddy currents are produced. These eddy currents are in phase with the induced E.M.F., and they will tend to oppose the changes in the magnetism. Other con-

ditions being similar, the mean demagnetising tendency of such currents will be directly proportional to the specific conductivity of the metal in the path of the magnetic lines; the power necessary to maintain these currents is dissipated in the form of heat in the mass of the metal, and this loss of power would be very considerable unless special precautions were taken to reduce it. The remedy—which is adopted in the cores of all alternating-current machinery—is to laminate or divide the metal core in a direction parallel to the magnetic flux, and slightly insulate the adjacent plates or wires from each other. In this manner the losses may be reduced to a very small amount.

The power dissipated in the laminated iron forming the cores of transformers and alternating-current machinery is proportional to the square of the E.M.F. induced in the local circuits in which eddy currents circulate; it is therefore approximately proportional to the square of the maximum value of the induction, and can be expressed by the formula

$$\text{watts per pound} = \frac{6}{10^9}(t f \mathrm{B})^2,$$

where t = thickness of laminations in inches (usually about ·014);
f = frequency in periods per second;
B = maximum value of the induction in gausses (C.G.S. lines per square centimetre).

23. **Hysteresis.**—The losses due to hysteresis occur only in the case of the magnetic lines passing through a *magnetic* metal, such as iron, and they are in no wise dependent upon the degree of lamination of the magnetic circuit.

It is well known that all iron, even the softest and purest, retains some magnetism after the magnetising force has been removed. By applying a magnetising force in the opposite direction this *residual* magnetism is destroyed, and the magnitude of this force—or, in other words, the amount of work which has to be done to withdraw this magnetism—depends upon the quality of the iron. Soft annealed wrought iron retains most magnetism; but, on the other hand, it parts with it more easily than the harder qualities of iron and steel, and for this reason requires the least expenditure of energy to carry it through a given cycle of magnetisation.

The frequency, or number of alternations of the magnetism per second, does not appreciably influence the loss through hysteresis *per cycle*, and, therefore, the loss of power per cubic inch or per pound of iron in the path of the magnetic lines will be proportional to the frequency. It has also been ascertained experimentally that the energy expended in carrying a given sample of iron through one complete cycle of magnetisation is approximately proportional to the 1·6th power of the limiting induction; and if B stands for the maximum value of the induction, we may, therefore, write

$$\textit{watts lost per pound of iron} \propto B^{1\cdot 6} \times \textit{frequency}.$$

In the case of well-annealed commercial iron stampings, the hysteresis loss per pound at a frequency of 50 periods per second would be about ·09 watt with a limiting induction of 15,000 C.G.S. lines *per square inch* of cross-sectional area, and about ·25 watt at double this induction—*i.e.*, 30,000 C.G.S. lines.

Assuming the iron stampings to be ·014 in. thick, and sufficiently well insulated from each other, the eddy-current losses for the same frequency and maximum

4

induction would be approximately ·02 and ·07 watt per pound; from which it will be seen that the losses due to eddy currents in the iron cores are of less importance than the hysteresis losses, and, indeed, with low inductions and low frequencies the eddy-current losses are almost negligible.

Although the losses through hysteresis vary considerably with different qualities of iron or steel, the following formula may be useful for approximate calculations. It gives the watts per pound for a good quality of transformer iron:

$$w = \frac{1 \cdot 3}{10^8} f B^{1.55}$$

—where B is the *maximum* value of the induction, in gausses or C.G.S. lines per square centimetre. To obtain watts lost per cubic inch, multiply by ·28.

In the case of silicon steel sheets, frequently used for transformer cores, the loss might be only 75 per cent. of the amount given by the above formula.

24. **General Conclusions regarding the Introduction of Iron in the Magnetic Circuit.**—Let us consider a choking coil consisting of a number of turns of wire, and draw two vector diagrams for such a circuit: the first (Fig. 19) showing relation of current to E.M.F. on the assumption that the solenoid or coil of wire has an air core—*i.e.*, no iron core—and the second (Fig. 20) showing what takes place in the same coil when an iron core is introduced for the purpose of reducing the current.

For the sake of comparison, we shall assume, in the first place, that the resistance, R, of the coil is constant in both cases, and equal to 4 ohms; and in the second place we shall see what are the relations of impressed

E.M.F. and current to induce in each case a back E.M.F. of 100 volts.*

In Fig. 19 (which should be compared with Fig. 11, p. 24) draw O E_1 of such a length as to represent the induced E.M.F. of 100 volts, and O I exactly 90 degrees *in advance*, to represent the magnetising current—that is to say, the alternating current of which the maximum value will produce the necessary magnetic flux to induce the E.M.F. of 100 volts in the windings of the coil. We shall suppose this current, I, to be exactly 10 amperes. The E.M.F., E_2, required to send this current against the resistance of 4 ohms will be

$$E_2 = 10 \times 4 = 40 \text{ volts},$$

and this component of the total E.M.F. must be drawn *in phase* with O I. The total necessary impressed E.M.F., E, is obtained by compounding O E_2 and an imaginary E.M.F. exactly equal, but opposite to O E_1. This will give us the value of E as about 108 volts, and the angle of lag $\theta = 68$ degrees.

It will be noticed that the current lags considerably behind the impressed E.M.F., and this lag would approach more and more nearly to its limiting value of 90 degrees if the resistance of the coil were diminished: if it were possible to make $R = 0$, the current I would be actually *wattless*; as it is, the power lost in the choking coil is all spent in heating the wire—the alternating magnetic flux

* The relation between magnetic flux and current was discussed in article 14; and in article 15 it was explained how the induced E.M.F. can be calculated for any particular point in the cycle of magnetisation. We shall, therefore, assume that the current required, in the present example, to produce the given induced E.M.F. of 100 volts, can be readily calculated; but if the reader wishes to go more fully into the question, he is referred to Appendix I. at the end of the book.

costs nothing to produce. The actual power lost in our example is 400 watts. It is equal to $I^2 R$ or to $O I \times O E_2$, or to $O I \times O E \times \cos \theta$.

We shall now suppose that the magnetic circuit, instead of being entirely through air, is made up either partly or

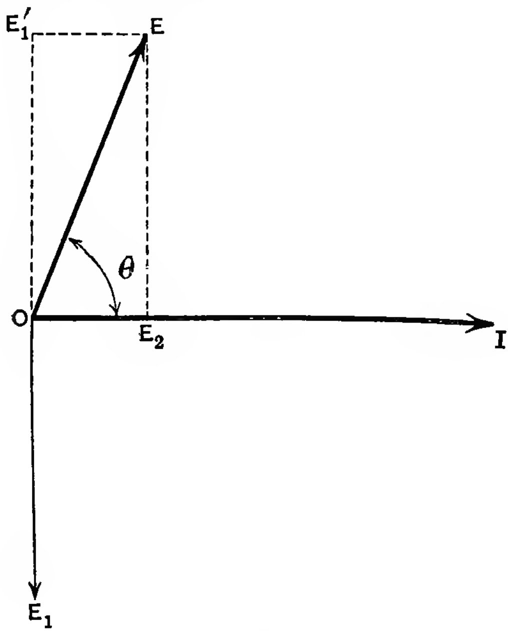

FIG. 19.

wholly of iron, which must be laminated to prevent an abnormal loss through eddy currents.

Draw $O E_1$ in Fig. 20, as before, to represent 100 volts; but $O I_m$—*i.e.*, the necessary magnetising current—will not be so great as O I in Fig. 19, because the introduc-

tion of the iron core has provided an easier path for the magnetism. We shall suppose that O $I_m = 5$ amperes, or half the magnetising current of the previous example. This is not, now, the *total* current in the coil, but merely

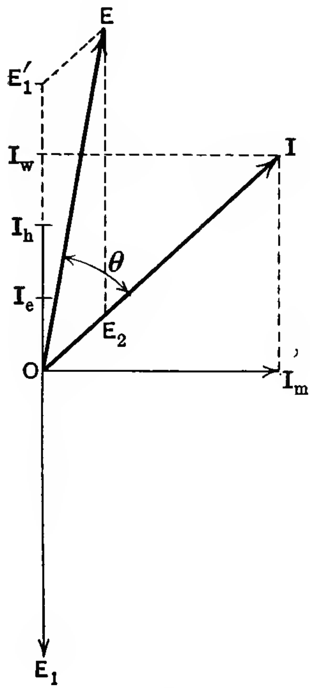

FIG. 20.

that component of the total current which is required to produce the necessary magnetic flux: it must be drawn, as before, exactly 90 degrees in advance of O E_1.

Let us suppose that the *eddy currents* in the iron are

responsible for a loss of power equal to 150 watts, and that the *hysteresis* is responsible for 300 watts. This total power of 450 watts has to be supplied by the generator—or whatever source of supply the choking coil is connected to—in the form of a component of the total current flowing *against* the induced E.M.F., the portion of the current required to provide for the eddy-current loss being $\frac{150}{100} = 1 \cdot 5$ amperes, while the hysteresis component of the current will be $\frac{300}{100} = 3$ amperes.

These two components of the total current are shown as $O\,I_e$ and $O\,I_h$ in Fig. 20, and they are drawn in a direction exactly *opposite* to $O\,E_1$ (the induced E.M.F.). The total current O I is obtained by compounding $O\,I_m$ and $O\,I_w$, the last vector being equal to the sum of the two energy components, $O\,I_e$ and $O\,I_h$.

As regards the E.M.F., the volts required to overcome the ohmic resistance of the coil are, as before, in phase with the total current O I; and since this measures very nearly 7 amperes, the vector $O\,E_2$ must be drawn of such a length as to represent $7 \times 4 = 28$ volts.

The resultant or total necessary impressed E.M.F. is obtained by compounding $O\,E_2$ and the imaginary E.M.F. $O\,E'_1$ exactly equal and opposite to $O\,E_1$. It will be found that the angle of lag, θ, is now about 38 degrees, while the total power lost $= O\,I \times O\,E \cos\theta =$ about 650 watts.

Thus, although the $I^2 R$ losses are less, owing to the reduction of current due to the introduction of an iron core, the losses in this core are themselves of such importance as to make the total losses greater than in the previous example. This, however, is by no means necessarily the case when iron is introduced in the mag-

netic circuit. The assumptions made for the purpose of drawing the above diagrams are arbitrary, but they serve the purpose of showing in what manner the losses in the iron core appear in the supply circuit and affect the magnitude and phase displacement of the total current.

25. **Capacity.**—The effects of electrostatic capacity are of far more importance in alternating-current than in continuous-current work, and we shall briefly examine the conditions arising out of the introduction of a condenser in a circuit carrying an alternating current.

The chief difference, in this respect, between a continuous—or unidirectional—and an alternating current is that the former, after charging the condenser, will be absolutely interrupted, while the latter will continue to flow to an extent depending upon the capacity of the condenser, the pressure, and the frequency (not to mention the wave form, which must also be taken into account).

Any arrangement of two conductors of electricity separated by an insulator forms a condenser, of which the capacity will be large or small depending upon the nearness, or otherwise, of the conductors, and the nature of the dielectric between them.

The effects of electrostatic capacity in the case of conductors supported on poles at a reasonable distance from the ground are very small, even on a long transmission line, *provided the frequency is low;* but further reference will be made to this particular case when treating of the transmission of power by polyphase currents. The capacity of an underground cable, especially when insulated with rubber, is a much more serious matter. The capacity per mile of any particular make and size of cable will always be furnished by the manufacturer, and the total capacity of any feeder or

distributing system can, therefore, be readily calculated when the length of the different sizes of cable is known.

26. **Current Flow through a Condenser.**—If two ballistic galvanometers are connected to the terminals of a condenser, in series with a battery or other source of continuous E.M.F., as shown in Fig. 21, it will be found, on depressing the key K, that the swing of both galvanometers is such as to indicate that the same quantity of electricity which passed into one set of plates has passed out of the other set of plates. But, nevertheless, the condenser is now *charged*, and if the key K_1 be depressed

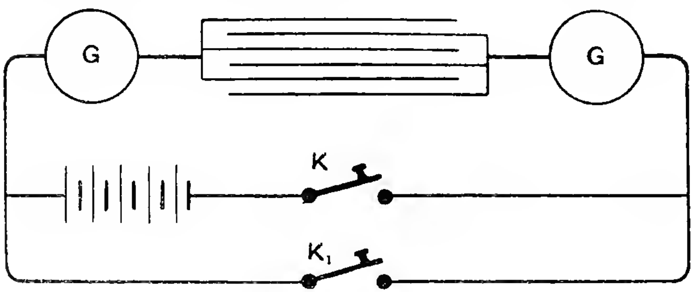

FIG. 21.

(after releasing the key K), the same quantity of electricity will flow through the circuit as when the charging key was depressed, only the flow will be in an opposite direction, as indicated by the swing of both galvanometers being approximately the same as before, but in the reverse direction.*

* A charged condenser should be considered, not as containing a store of electricity, but a store of electric energy, which may be used for doing useful work by joining the terminals of the condenser through an electric circuit. In this respect it may with advantage be compared with a deflected spring which, so long as it is kept in a state of strain, has a certain capacity for doing work.

Bearing these facts in mind, it will not be difficult to understand that an alternating current will have the property of flowing through a condenser, and the following remarks should assist the reader in forming a mental picture of what takes place

In Fig. 22, A is an alternator and C a condenser, which, in practice, might be a long concentric cable with the two conductors disconnected and well insulated at the distant end

So long as the alternating current is flowing in a positive direction, the condenser is being charged, but as

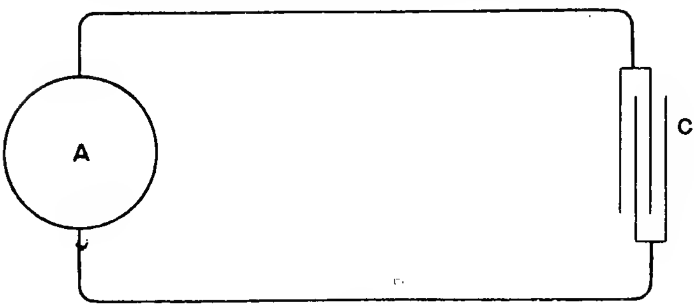

FIG. 22.

soon as ever the current reverses, the condenser begins to discharge. The maximum charge of the condenser—which corresponds with the maximum value of the condenser E.M.F.—will, therefore, of necessity occur at the instant when the current is *changing in direction.*

In Fig. 23, the curve I represents the current flowing through the condenser, and the dotted curve Q represents the charge (in coulombs), which, it will be noticed, has been steadily growing from its *negative* maximum value to its *positive* maximum value (*d*) during the whole time

the current has been flowing in a positive direction. This curve Q will therefore lag behind the current curve by exactly a quarter of a period.

Next, as regards the condenser E.M.F.; it is evident that this will always be such as to *oppose* the applied

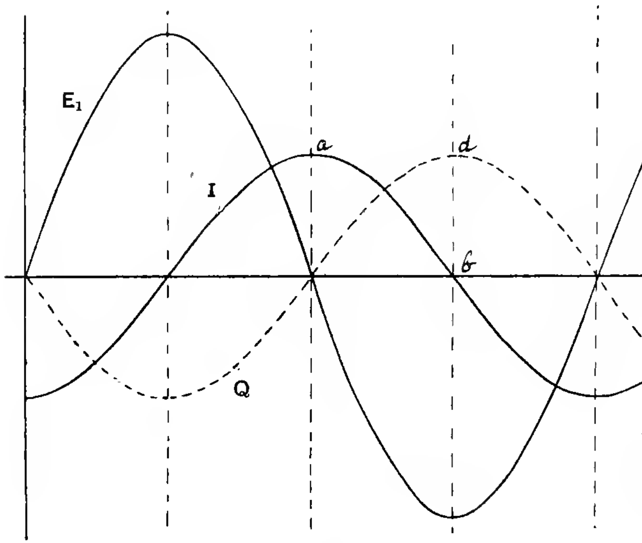

FIG. 23.

E.M.F., or, n other words, it will tend to *expel* the charge.

We may, therefore, draw the curve E_1 exactly opposite to Q, the length of the ordinate at every point being equal to

$$\frac{\textit{quantity (or charge of electricity)}}{\textit{capacity of condenser}},$$

If we neglect the resistance of generator and leads, and also certain losses which occur in the condenser itself, the curve E_1 in Fig. 23—*i.e.*, the potential difference at condenser terminals—will be exactly equal and opposite to the alternator E.M.F. Thus, the *condenser E.M.F.* is exactly a quarter of a period *in advance* of the current; and this is interesting when it is remembered that the *E.M.F. of self-induction* always *lags behind* the current by the same interval.

This suggests the possibility of counteracting the effects of self-induction by the introduction of capacity, or *vice versa;* and, indeed, the one is sometimes used in practice for the purpose of partially neutralising the effect of the other, with moderate success; variations in the frequency or any slight modification in the wave form of the applied E.M.F., such as will occur as the load on the generator is varied, will considerably interfere with the balancing action of a condenser and choking coil.

With regard to the magnitude of the condenser current; let C be the capacity in *farads* of a condenser with an alternating E.M.F. of maximum value $E_{max.}$ volts between the two sets of plates. The total charge in coulombs, at the end of one half-period, will be $C \times E_{max.}$ But *quantity = current × time;* hence the maximum charge in coulombs is

$$Q = I_m \times \frac{1}{4f},$$

where I_m is the mean or *average* value of the current, and $\frac{1}{4f}$ is the time, in seconds, during which the current changes from maximum to zero value. Thus, by referring to Fig. 23, it will be seen that, while the current falls from its maximum at *a* to zero value at *b*, the charge

has grown from zero value to its maximum value at *d*. We can therefore write

$$C \times E_{max.} = I_m \times \frac{1}{4f}$$

or

$$I_m = 4 f C E_{max.}$$

The quantity we generally require to know being the R.M.S. value of the current, it is necessary to multiply the above quantity by the *form factor*, or ratio $\frac{\text{virtual value}}{\text{mean value}}$ of the current. If the applied E.M.F. is sinusoidal, the current wave will also be a sine curve, and the form factor is then $\frac{\pi}{2 \times \sqrt{2}}$ or 1·11, and

$$I_c = \frac{2\pi}{\sqrt{2}} f C E_{max.}$$

If E stands for the virtual or R.M.S. value of the E.M.F. (obtained by dividing the *maximum* value by $\sqrt{2}$), the formula becomes,

$$I_c = 2 \pi f C E.$$

The capacity is usually stated in microfarads, and if C is so expressed, the right-hand side of the equation must be divided by 1,000,000.*

* It is important to note that the capacity current will depend considerably upon the wave form of the applied potential difference. Thus, in place of the multiplier 2π (or 6·283), in the formula for a simple harmonic variation—or sine wave—we should have to substitute $4\sqrt{3}$ (or 6·928) if the wave were of a triangular shape, with straight sides, and this corresponds to an increase of 10 per cent. in the amount of the capacity current. A peaked form of wave is, therefore, to be avoided if it is desirable to keep down the capacity

In very long underground feeders, the current flowing into the cable when there is no connection at the distant end is sometimes considerable, especially when the cables are insulated with rubber, and the frequency and pressure are high. This is the capacity current, and its effect in increasing the total current is readily ascertained when it is remembered that this component of the total current will be 90 time-degrees out of phase with the useful current delivered at the distant end of the feeder, if we assume the load to be non-inductive.

Thus, if

I = the ingoing current;
I_1 = the condenser current (say 10 amperes);
I_2 = the outgoing current (say 60 amperes),

we may write

$$I = \sqrt{I_1^2 + I_2^2} = 60{\cdot}82 \text{ amperes},$$

which is very little greater than the outgoing current; and the example illustrates the fact of the capacity current becoming of less and less importance as the load on the feeder increases.

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current; but since the wave form of all alternators changes, to a certain extent, as the load comes on, there will, in any case, be a variation in the capacity current depending upon the load, and this variation may even amount to as much as 50 per cent. It is, however, the capacity current at *light loads* which is of the greatest importance in practice, because at times of heavy load its effects are not felt to anything like the same extent. Should the E.M.F. wave have several peaks—*i.e.*, more than one maximum value per half wave—this would be quite the worst form of wave as regards the capacity current, which might easily be doubled, solely on account of this peculiarity in the shape of the wave.

Perhaps one of the most extensive systems of concentric, rubber-insulated, underground cables is that of the Société Anonyme Belge Éclairage Électrique de Saint Pétersbourg. There are about 240 kilometres of cable in St. Petersburg forming the high-tension network of this company's mains. The charging current, for the frequency of 42·5 and a pressure of 2,000 volts, is about 47 amperes; but there are nearly 1,000 transformers on the circuits, taking a total magnetising current of about

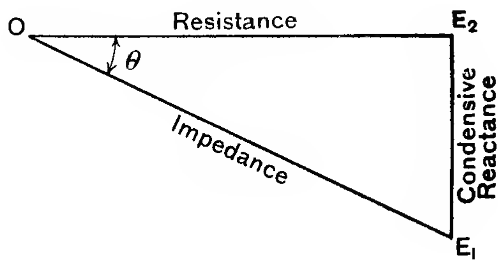

FIG. 24.

250 amperes, which more than balances the abnormally large capacity current.

27. **Condensive Reactance.** — Just as inductive reactance can be expressed in ohms (see article 17, p. 40), so also can we express the reactance due to the capacity of a circuit, provided always that the connection between frequency and reactance is not lost sight of.

Reactance is a quantity which, when multiplied by the current, will give that component of the total E.M.F. in the circuit which is necessary to balance the counter E.M.F. of self-induction or of capacity, as the case may be. It may therefore be thought of as the counter

E.M.F. in the circuit when unit current is flowing; and when represented by vectors, it must always be drawn at right angles to the current vector. But whereas the vector representing the balancing component of the inductive reactance is drawn in *advance* of the current vector (see Figs. 17 and 18), the corresponding vector representing condensive reactance must be drawn in the direction of retardation—*i.e.*, 90 time-degrees *behind* the direction of the current vector.

In Fig. 24, O E_2 is the *resistance* of the circuit, which may also be considered as the voltage component in phase with the current, *divided* by the current.

E_2 E is the *condensive reactance*, of which the value in ohms, on the sine wave assumption, is—

$$X = \frac{1}{2\pi f C},$$

since it may be considered as the voltage component required to balance the condenser E.M.F., *divided* by the current.

O E is the *impedance*, of which the value, in ohms, is—

$$Z = \sqrt{R^2 + X^2},$$

and it may be considered as the total E.M.F. impressed on the circuit, *divided* by the current.

The reason why condensive reactance may be considered as the exact opposite of inductive reactance should now be clear to the reader; because if the circuit were supposed to have inductance as well as capacity, the reactance due to the choke coil effect would be a continuation of the line E E_2 of Fig. 24, but it would be drawn *above* the line O E_2; and the complete formula for

the impedance of a circuit, taking into account resistance, self-induction, and capacity, is—

$$Z = \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

—where $\omega = 2 \pi f$, on the sine wave assumption.

CHAPTER III

POLYPHASE CURRENTS—GENERAL PRINCIPLES AND SYNCHRONOUS GENERATORS

28. WHEN single-phase alternating currents were first used, on a large scale, for incandescent electric lighting, no satisfactory single-phase motors were obtainable. One of the difficulties in the way of producing commercial alternating-current motors was, no doubt, the high frequency (about 100 in this country) which had been adopted for the lighting circuits; but, apart from this, the problem of designing single-phase motors capable of starting under load is not a simple one. The introduction of polyphase currents, by the aid of which a *rotating magnetic field* could be obtained from stationary windings, completely solved the problem of the alternating-current motor.

The idea of producing a rotating magnetic field by means of polyphase currents may be said to date as far back as 1879, when Walter Baily read a paper before the Physical Society, entitled "A Mode of Producing Arago's Rotation"; but, although several patents were taken out during the ensuing years, nothing serious was done in this connection until 1891, when M. von Dolivo-Dobrowolsky and Mr. C. E. L. Brown put down their complete scheme of three-phase power transmission from

Lauffen to Frankfurt. After this, the use of polyphase motors extended rapidly.

Definition.—A polyphase current may be defined as a combination of two or more alternating currents, all of the same periodicity, but having certain phase differences between them.

29. **Production of Rotary Field.**—In Fig. 25 we have a representation of a two-phase current, the phase

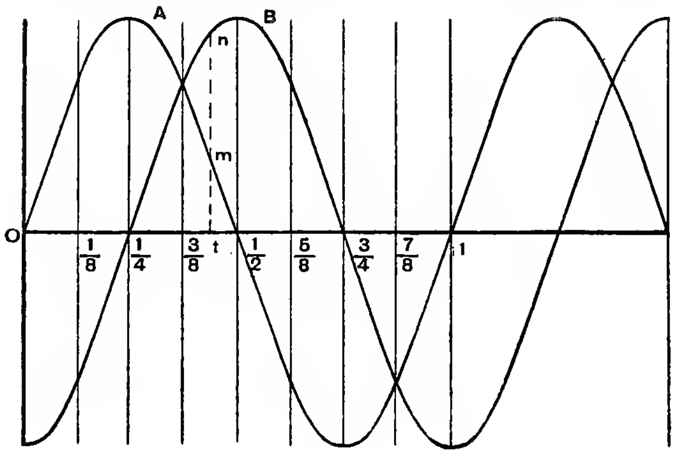

FIG. 25.

angle of which is 90 degrees. It will be seen that this diagram merely represents two simple harmonic alternating currents, A and B, of the same frequency and the same amplitude, but differing in phase by a quarter of a period.

Let us imagine these two currents to be flowing respectively in two coils, A and B, set at right angles to each other, in the manner shown in Fig. 26. We shall

further assume that when the current A is flowing in a positive direction—*i.e.*, when it is measured *above* the datum line, Fig. 25—it will produce a magnetic field in the upward direction through the coil A; and when the

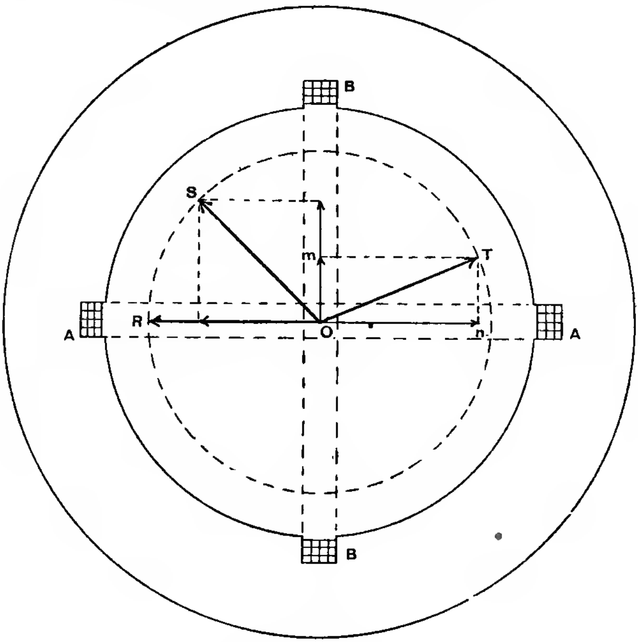

FIG. 26

current B is positive, it will produce a field from left to right through the coil B.

In order to see clearly how the resultant field, due to the currents in the two coils, revolves round the centre, O (Fig. 26), let us start at the instant O (Fig. 25). The resultant field at this instant is evidently O R, which

is the vector representation of the field due to the maximum negative value of the current B—the value of A, at this instant, being zero.

After the lapse of one-eighth of a period, the two currents (on the assumption of a sine curve wave form) are exactly equal, but A will be positive and B negative: the resultant field will be O S. After another eighth of a period, the resultant field will be that due to the maximum positive value of A; it would be represented by a vector equal in length to O R, but drawn in a vertical direction from the centre, O, upwards. At the instant t (corresponding to the 7/16ths of a complete period) $A = t\ m$ and $B = t\ n$, both being positive. The combination of these two component fields gives us O T as the resultant. It is not necessary to carry the construction any further, for it is evident that the combined effect of the two currents will be to produce a magnetic field of constant intensity, equal to that due to the maximum value of either current, revolving round O at the uniform rate of one revolution in the time of one complete period.

Although we have assumed a phase difference of a quarter period between the two currents, and an angle of 90 degrees between the axes of the two coils, it should be stated that a rotating field can be obtained by means of two phase currents, whatever may be the phase difference between them.

The necessary condition for the production of a uniformly rotating field of constant strength by the combination of two alternating magnetic fields is simply that the angle which the positive directions of the alternating fields make with each other must be the *supplement* of the phase angle.

Thus, if we had assumed a phase difference of

100 degrees instead of 90 degrees between the two currents shown in Fig. 25, we should have found it necessary to draw the coils A and B in Fig. 26 with an angle of $(180 - 100)$ degrees, or 80 degrees between them instead of at right angles to each other.

With regard to the intensity of the resulting rotating field, on the assumption of the sine law of variation, this will be equal to the maximum value of either of the two (equal) alternating fields, *multiplied by the sine of the phase angle.*

Although the proof of the above statements is simple, we shall content ourselves with an illustration, which, at the same time, will explain the use of another diagram, sometimes preferable to the curves of Fig. 25, for representing alternating quantities which follow the simple harmonic law of variation.

We shall assume the phase difference between the two currents to be one-third of a period, instead of one-quarter of a period, as in the previous example.

Draw O A and O B, in Fig. 27, to represent the *maximum values* of the two alternating magnetic fields. These vectors must be drawn with an angle of 120 degrees between them to represent the phase difference of one-third of a period.

With O as a centre, describe the dotted circle of radius equal to O A or O B, and on each end of the vertical diameter describe a smaller circle, of radius equal to half O A or O B, in the manner shown in the figure.

Now imagine the two vectors to revolve in a *counter-clockwise direction*—as indicated by the arrow—at a uniform rate of one revolution in the time of one complete period. The length of either vector, measured from the centre, O, to the point where it is cut by one of the inner circles will then be a measure of the field intensity due to the

current in phase A, or in phase B, as the case may be, at any particular instant throughout the complete cycle. Measurements made in the upper circle would denote, say, a positive direction of the magnetic field, while

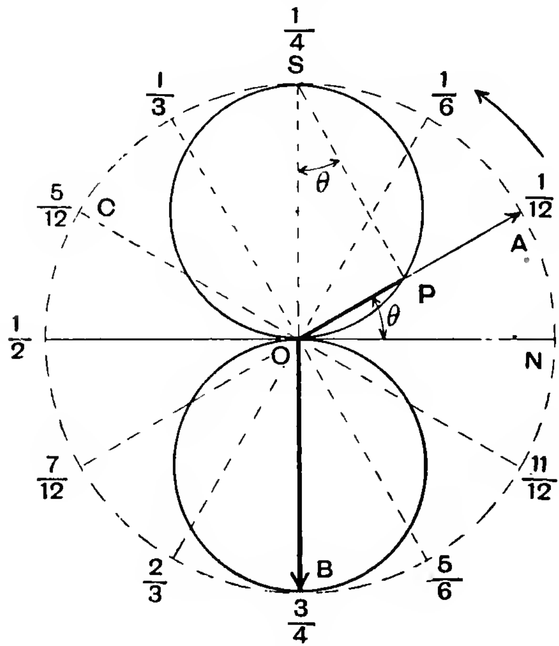

FIG. 27.

measurements made in the lower circle would indicate a magnetic flux in the opposite, or negative, direction.

The proof of the above statement is very simple. In the first place, the angle O P S of the triangle O P S, constructed within a semicircle of which O S is the diameter, is always a right angle; and since the angle

P S O is equal to A O N, we see that O P is always equal to O S sin θ, or O A sin θ, which is the same thing. Now, the reason why the simple harmonic curves as shown in Fig. 25 are also called sine curves, is simply because the ordinates of such curves are directly proportional to the sine of the time-angle; and it, therefore, follows that the portion, O P, of any revolving vector such as O A or O B, which is contained within either of the two small circles, will be a measure of the instan taneous value of an alternating quantity which follows the simple harmonic law of variation.

Let us now see how the combination of the two alternating fields, A and B—when B lags behind A by one-third of a period (120 degrees)—will produce a rotating field when combined at an angle of $(180-120)$ or 60 degrees.

In Fig. 28, the two coils are shown at an angle of 60 degrees to each other; and, since the field due to each coil will be at right angles to the plane of the coil, the field component due to coil A will aways lie on the diameter 10—4, while the field component due to B will lie on 12—6.

At the particular instant corresponding to the position of the vectors shown in Fig. 27, the resultant field will be O—1, obtained by compounding the vectors O *c* and O *d*. After an interval of a twelfth of a period (30 degrees), the two components will be equal, but of the same sign as before—*i.e.*, A will still be positive, and B negative: the resultant field is O—2, of which the components are O—12 and O—4. After another twelfth of a period, the vector A (Fig. 27) will have reached its maximum positive value, while B will have fallen to half its maximum negative value: the resultant field will be O—3, obtained by compounding O *e* and O *f*. And so

on: it will be found that the resultant field will revolve round O at a uniform rate of one revolution per period, and it will have a constant value represented by the radius of the dotted circle in Fig. 28. This radius is evidently equal to O c × sine 60 degrees; that is to say,

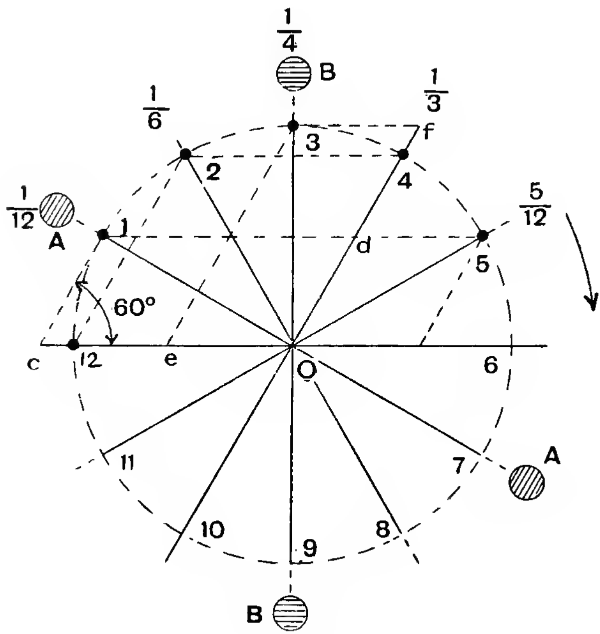

Fig. 28.

the intensity of the uniform rotatory field will—for this particular phase difference—be equal to the maximum value of any one of the component fields multiplied by the sine of 60 degrees.

This confirms the general statement previously made,

to the effect that the resulting rotary field is equal to the maximum value of either of the two equal alternating fields, multiplied by the *sine of the phase angle.* It is true that the phase angle, in this example, is 120 degrees, and not 60 degrees, but the sine of 120 degrees is the same as sine (180 – 120) degrees or 60 degrees.

30. **Three-Phase Currents.** — It has been shown how a rotating field may be produced by means of two-

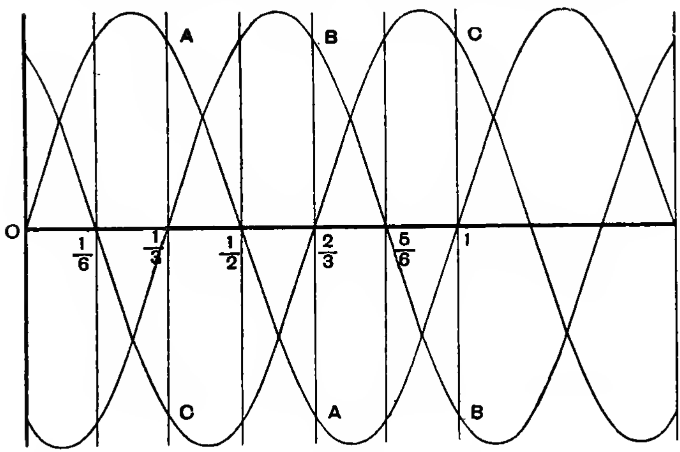

FIG. 29.

phase currents, and the reader should, therefore, have no difficulty in understanding that the same results can be obtained by using any number of currents, with suitable phase differences between them, and a corresponding number of field coils arranged with the proper angular spacing between them.

Fig. 29 shows the three currents of a three-phase system with a phase angle between them of 120 degrees,

or one-third of a complete period. It is not proposed to draw any more diagrams on the lines of Figs. 27 and 28; but it will be understood that we have merely to imagine a third vector drawn in Fig. 27 in the direction O C, and a third coil in Fig. 28 across the diameter 11—5, when it will be found that the combination of the three alternating fields will produce a uniform rotating field, exactly as in the case of the two-phase system; the only difference being that the intensity of the rotating field will now be equal to 1·5 times the amplitude of any one of the three (equal) alternating fields.

In practice the rotating field is not necessarily of constant strength, neither does it always revolve at a uniform rate. Thus, if the currents in the coils do not follow the sine law, the resultant field will vary in amount.

Again, if the induction in the iron forming the magnetic circuit be carried to high values, the *permeability* may be far from constant, and, even with a current following the simple harmonic law, the magnetism might vary in a very different manner. It is, therefore, advisable to work with comparatively low inductions, and to use generators giving, as nearly as possible, sine curve E.M.F. waves under all conditions of load.

The effect of inaccurate spacing of the coils, or of unequal currents in the different phases, will also be to produce distortion of the resulting field; but the remedy is obvious.

31. **Utilisation of Rotary Field.**—Consider a small continuous-current dynamo, of which the field magnets are fully excited, but with the brushes lifted off the commutator.

If there is no fault in the armature winding, it will be possible—by means of a lever or pulley fixed to the shaft—to revolve the armature by hand; because, although

E.M.F.s will be generated in the windings, these balance each other, and there will be no circulating current.

If, on the other hand, one or several of the armature coils are short-circuited, the effect will be the same as if a strong brake had been applied, and it will not be possible to revolve the armature by hand, except at a very slow rate. If only one coil be short-circuited, it will be possible to move the armature freely so long as the shorted conductors lie in the gap between the poles; but as soon as they enter the fringe of magnetism under the pole tips, the braking effect will be felt. Again, if all the coils are shorted—a result which may be attained by winding a few turns of bare copper wire round the commutator—there will be no positions of the armature between which it can be freely moved; but the magnetic braking action will be felt uniformly throughout the complete revolution of the armature.

It is evident that these effects are due to the currents generated in the closed coils of the armature, for these will always flow in such a direction as to oppose the motion of the conductors through the field. The torque which has to be exerted to make the armature revolve will be proportional to the product of field strength and armature ampere turns under the pole-pieces, and the power required to revolve the armature will be expended in the form of $I^2 R$ losses in the armature windings.

Let us now suppose that the whole framework of the dynamo, including the system of field magnets, instead of being bolted to solid foundations, is free to revolve on the armature spindle. We shall not, with this arrangement, experience the same difficulty in revolving the armature, because, if the system of (excited) field magnets be perfectly balanced, and the friction of the bearings be small, the revolving armature will draw the

field magnets round with it; and, indeed, if frictional losses could be entirely neglected, the armature and field would be magnetically locked together, while the whole system could be revolved in space without requiring any expenditure of power.

If there is appreciable frictional resistance to the motion of the system of field magnets, the latter will revolve at a slightly slower speed than the armature; that is to say, there will be a certain *slip*, or *relative speed* between the two parts, which will automatically adjust itself until the currents generated in the armature conductors are sufficient to produce the necessary torque.

Although we have briefly considered the case of a revolving armature dragging the field with it, the arrangement can be reversed, and if the system of field magnets be revolved, the short-circuited armature will be dragged round at approximately the same speed: the E.M.F.s generated in the armature windings, for a constant strength of field, will depend upon the *rate at which the magnetic lines are cut by the armature conductors*—*i.e.*, upon the *relative* speed of field and armature.

In a polyphase motor, the armature, or *rotor*, may consist of three or more windings *closed upon themselves*, or—as is frequently the case in small machines—it may be of the squirrel-cage type, in which heavy conductors, threaded through the iron stampings near the periphery, are joined together at each end by substantial metal rings. In both cases the effect is the same: the revolving field—produced in the manner already described, by the windings on the *stator*—generates E.M.F.s in the short-circuited rotor windings, and the resulting current produces a torque tending to drag the armature round against the resisting forces; these resisting forces being made up of bearing friction, windage, hysteresis, and eddy-current

losses when running light, with the addition of the torque due to the load on the motor when the latter is doing useful work.

Although the theory of the polyphase motor will be treated more fully in a later chapter, it is important that the reader should have a clear mental picture of the revolving field dragging the short-circuited armature with it. Imagine the resistance of the closed armature windings to be *nil;* then, whatever may be the load on the motor, its speed would be absolutely constant; the armature would be magnetically locked with the revolving field, because the slightest "slip," or difference in speed between field and armature, would induce infinitely great currents in the windings of the latter, and instantly pull it into step.

In practice the *slip* does not exceed 5 per cent., and it will reach its greatest value when the motor is doing its maximum load. It may be asked, what does this 5 per cent. difference of speed represent? The answer is that it represents power lost in heating the armature conductors. Thus, if the field revolves at 1,000 revolutions per minute and the armature at 960 revolutions, and if the torque at this particular speed is 100 statical foot-pounds, we have:

Horse-power *imparted to armature*

$$= \frac{100 \times 2\pi \times 1{,}000}{33{,}000}$$
$$= 19{\cdot}05$$

Horse-power *developed by armature*

$$= \frac{100 \times 2\pi \times 960}{33{,}000}$$
$$= 18{\cdot}3$$

hence, horse-power wasted in $I^2 R$ losses in armature conductors

$$= \cdot 75$$
$$= 560 \text{ watts.}$$

A thorough understanding of how the current in the primary circuit (stator windings) grows in response to the demand for additional power as the load on the rotor increases is not possible without a knowledge of the main principles underlying the working of alternate-current transformers: we shall, therefore, leave the more detailed consideration of the induction motor for the present, and briefly deal with the question of polyphase generators.

32. **Polyphase Generators.**—It is not intended, in these pages, to give formulæ or data which would be of use to those engaged in the design of polyphase generators; but an attempt will be made to give a clear explanation of the elementary principles involved in the generation of two- or three-phase currents.

In nearly all large alternators or polyphase generators, the system of field magnets revolves, while the armature is stationary. With this arrangement only two collecting rings are required, for conveying the exciting current to the field coils; the armature current being delivered from stationary terminals. The insulation of these terminals, together with that of the armature coils, is more easily carried out than if they formed part of the movable portion of the machine.

For low pressures, such as 200 volts, especially if the output is small, a design of machine with revolving armature will generally be found to be cheap and efficient.

Nearly all alternating-current generators are of the multipolar type—that is to say, they are provided with more than one pair of poles. A few machines, when

driven at high speeds by steam-turbines, may have only two poles, but these are the exception.

If p is the number of poles, and R is the speed in revolutions per minute, then

$$Frequency = \frac{R}{60} \times \frac{p}{2}$$

and in the case of large machines, running at comparatively low speeds, a fairly large number of poles will be found necessary to give the desired frequency.

Whatever may be the type of machine, or number of poles, we may consider the armature conductors to be cut by the magnetic lines in the manner indicated in Fig. 30. Here we have a diagrammatic representation of single-phase, two-phase, and three-phase windings. In each case, the system of alternate pole-pieces is supposed to move across the armature conductors in the direction indicated by the arrow. It will be noted that the conductors of each phase are shown connected up to form a simple wave winding; but this is only done to simplify the diagram, and it will be readily understood that each coil may contain a number of turns, attention being paid to the manner of its connection to the succeeding coil, in order that the E.M.F.s generated in the various coils shall not oppose each other.

The upper diagram shows a single winding, in which an alternating E.M.F. will be generated. In the middle diagram there are two distinct windings, A and B, so arranged relatively to each other and the pole-pieces that the complete cycle of E.M.F. variations induced in A will also be induced in B, but after an interval of time represented by a quarter of a period. This diagram shows the positions of the poles at the instant when the E.M.F. in A is at its maximum, while in B it is passing

through zero value. From these two windings we can, therefore, obtain two-phase currents with a phase difference of 90 degrees.

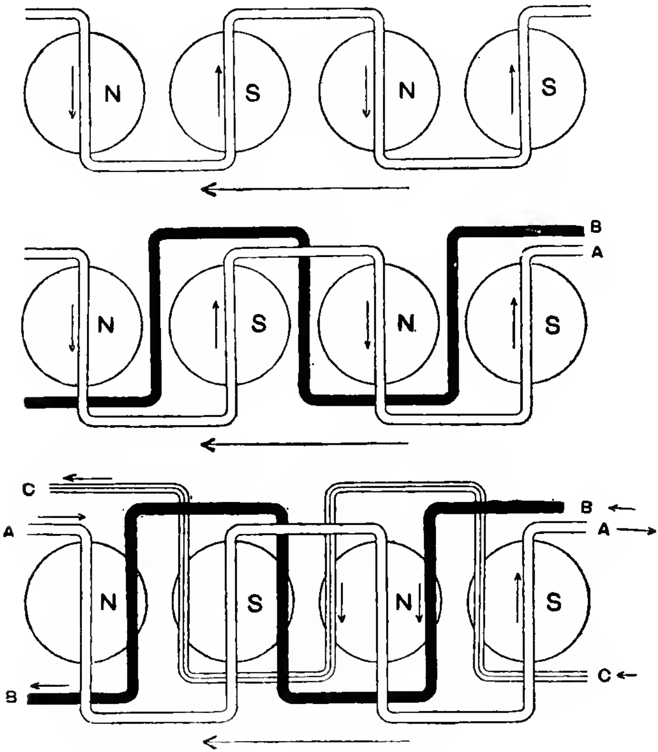

FIG. 30.

In the bottom diagram, the arrangement of three windings is shown, from which three-phase currents can be obtained, having a phase angle of 120 degrees

or one-third of a period between them. It will be noticed that, at the instant indicated by the relative positions of coils and poles, the E.M.F. in A is at its maximum, while in B and C it is of a smaller value, and in the opposite direction.

33. **E.M.F. induced in Generator Windings.**—With the assistance of the top diagram of Fig. 30, it will readily be seen that the total number of magnetic lines cut by any one armature conductor *per revolution* will be $N \times p$, where N stands for the total magnetic flux passing into the armature from any one pole, and p is the number of poles.

If Z be the total number of active conductors, counted round the circumference of the armature, and R stands for the speed in revolutions per minute, the *mean* E.M.F. in volts generated in any one winding, if all the conductors are connected in series, will be

$$E_m = Np \times Z \times \frac{R}{60} \times \frac{1}{100,000,000} \quad . \quad . \quad *$$

We are not generally concerned with the *mean* value of an alternating quantity; but it is not always possible to estimate very accurately the *shape* of the induced E.M.F. wave, which will determine the relation of the $\sqrt{\text{mean square}}$ value of this E.M.F. to its mean value. This will depend upon the disposition of the windings, the shape and spacing of the poles, etc. In order to obtain the actual volts at terminals on open circuit, we shall have to include a multiplier, k, which will depend upon the *form* of the wave. This multiplier, which is the ratio of the $\sqrt{\text{mean square}}$ value to the mean value of the induced E.M.F., is called the *form factor*. The general

* See article 15, Chapter II.

expression for the open-circuit terminal pressure therefore becomes:

$$E = k\,N\,p\,Z\,\frac{R}{60} \times 10^{-8}.$$

If the E.M.F. wave is a sine curve, $k = 1{\cdot}11$.

If the wave were rectangular in shape (which would not be possible in practice), the multiplier k would be unity.

Other calculated values of k are:

For triangular shape ... ... $k = 1{\cdot}16$.
For semi-circle or semi-ellipse $k = 1{\cdot}04$.

34. **Connections of Polyphase Armature Windings.**—Consider the armature winding of an ordinary continuous-current two-pole dynamo. If we imagine the commutator of such a machine to be entirely removed, the winding—whether the armature be of the drum or ring type—will be continuous, and closed upon itself. If the armature be revolved between the poles of separately excited field magnets, there will be no circulating current in the windings, because the magnetism which passes out of the armature core induces an E.M.F. in the conductors exactly opposite and equal in amount to that induced by the entering magnetism.

If we now connect to a couple of slip rings two points of the winding from the opposite ends of a diameter, the machine will be capable of delivering an alternating current. If we provide three slip rings, and connect them respectively to three points on the armature winding distant from each other by 120 degrees, the machine will become a three-phase generator.

In this manner, polyphase currents of any number of phases can be obtained, and if the windings, polar spaces, etc., are symmetrical, there will be no circulating current.

This method of connecting up the various armature coils of a polyphase generator is known as the *mesh* connection. In the case of three-phase currents it is also referred to as the *delta* connection.

The diagram, Fig 31, shows the three equidistant tappings from armature winding to slip rings required to produce three-phase currents. It is evident that the potential difference between any two of the three rings

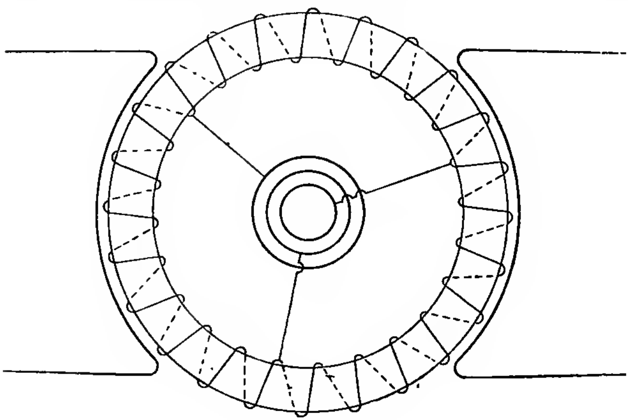

FIG. 31.

will be the same, since each section of the winding has the same number of turns, and occupies the same space on the periphery of the armature core. Moreover, the variations in the induced E.M.F. will occur successively in the three sections at intervals corresponding to one-third of a complete period.

Incandescent lamps may be connected across one or all three phases, as shown in Fig. 32. This lamp load is practically non-inductive; and we may, therefore, consider

the current to be in phase with the potential difference across the lamp terminals.

Let the vectors O I_1, O I_2, and O I_3, in Fig. 33, represent the loads on the three sections of the three-phase generator. They are drawn at an angle of 120 degrees to each other, because—being in phase with their respective E.M.F.s—they must necessarily differ in phase by a third of a period.

Referring to Fig. 32, we see that the resultant currents,

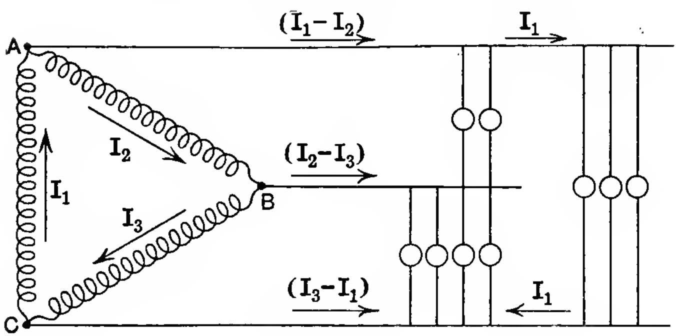

FIG. 32.

at the three terminals of the generator, which flow into the external circuit are:

At the terminal A	... ...	$I_1 - I_2$
,, ,, B	... ...	$I_2 - I_3$
,, ,, C	... ...	$I_3 - I_1$

It is, therefore, an easy matter to draw the vectors O A, O B, and O C in Fig. 33: these represent, by their length and phase relations, the currents in the three conductors leading to the lamp load.

It will be found that any one of these three vectors

always balances the other two; that is to say, any one vector, such as B, will be found to be exactly equal but opposite in direction to the resultant of A and C. That this is necessarily the case is evident when it is realised that, at every instant, the total of all currents leaving the

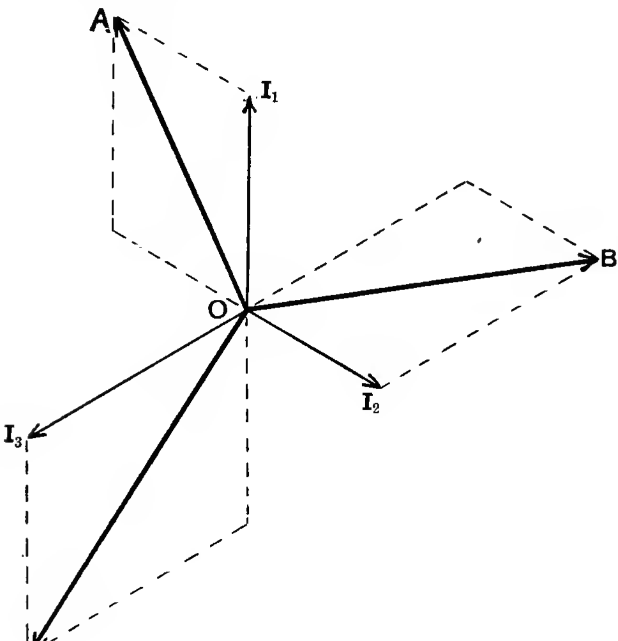

FIG. 33.

armature must be equal to the total of all currents returning to the armature, which is merely another way of saying that the sum of the three currents, A, B, and C, must be zero.

If the load is balanced, which will be the case if there

is an equal number of lamps on each of the three sections, or if the load consists, not of lamps, but of induction motors, then the three currents, I_1, I_2, and I_3, will be equal, and the currents leaving the terminals will also be equal, but greater than the armature currents: it can easily be shown that, for this condition of a balanced load, any one of the line currents is equal to the current in any one of the armature sections, multiplied by 2 cos 30 degrees, or by $\sqrt{3}$. Thus

$$\begin{aligned} O\,A &= I_1 \sqrt{3} \\ &= 1{\cdot}732\ I_1. \end{aligned}$$

Nature of Circulating Current in Practice.—Although there is, theoretically, no circulating current in the mesh connected armature windings of a polyphase generator, the conditions of practice are such that perfect balance is rarely, if ever, attained, and currents will, therefore, circulate in the windings in addition to those delivered to the mains. Such currents do not necessarily involve any appreciable loss of efficiency, but they must evidently increase, to a certain extent, the $I^2 R$ losses in the armature conductors.

The circulating current with the field magnets fully excited, but with open external circuit, would be a small percentage of the normal full-load current in a well-designed machine; at the same time, if there is want of symmetry in the arrangement of armature coils, or in the spacing and shapes of the pole-pieces, this circulating current might amount in practice to as much as 25 per cent. of the full-load current. A little consideration will make it clear that the function of this internal armature current is to maintain the potential differences at the three terminals such that their sum is, at every instant,

equal to zero. The direction of this current at every instant will be such as to react upon the pole-pieces, and so produce the necessary correction in the distribution of the magnetic lines entering the armature. The frequency of the circulating current will be three times that of the line current, because, while a current of the fundamental frequency cannot circulate in a delta-connected winding, the *third harmonic* and its multiples can circulate freely. One interesting conclusion to be drawn from these considerations is that the amount of the circulating current will depend upon the strength of the field, or on the magnetic induction. In other words, if an ammeter were inserted in series with the closed armature windings, it would be found to indicate an approximately steady reading notwithstanding large variations in the *speed* of the machine, *provided the excitation of the field magnets remained constant.* If the speed were kept constant, the amount of circulating current would be approximately proportional to the volts at terminals.

Star Connection of Three-Phase Armature Windings.—In practice this method sometimes has advantages over the mesh or delta connection. It consists merely in joining together the starting ends of the three armature windings in one common junction, or *neutral point*, and taking the finishing ends to the three terminals. It is not necessary to run a common return connection back from the load to this neutral point; each arm of the star connection forms the return path for the currents flowing in the other two arms.

This method of connecting up the coils will be again referred to when dealing with the question of transmission of power by three-phase currents; but if we refer back to Fig. 33 and imagine the vectors $O\,I_1$, $O\,I_2$, and $O\,I_3$ to stand, not for the *currents* in a mesh-connected

armature, but for the E.M.F.s in the three sections of the star connection, then the vectors O A, O B, and O C will represent the potential differences between the three terminals of the generator.

A simpler construction is shown in Fig. 34. Here $o\,e_1$, $o\,e_2$, and $o\,e_3$, represent the E.M.F.s in the three sections of the star-connected armature, and the sides of the triangle obtained by joining the ends of these three vectors correctly represent—both by their lengths and

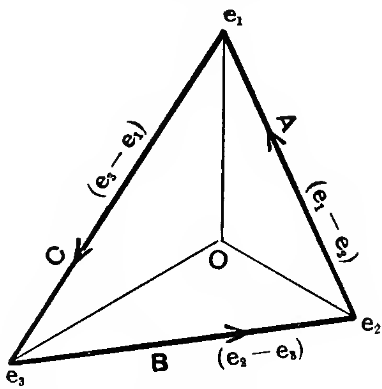

FIG. 34.

their directions—the amounts and phase relations of the pressures as measured between terminals.

If, as is usually the case, the three armature sections have the same number of turns, the triangle of vectors representing the terminal pressures will be equilateral, and—as explained in connection with the resultant *current* of a *mesh*-connected armature—the *volts* measured across the terminals of a *star*-connected machine will be $\sqrt{3}$ times greater than the volts measured across any one of the three (equal) armature sections.

Except for the possible advantage of being able to earth the neutral point of the star-connected winding, it is of little consequence to the user whether his machines are star or mesh connected: he is merely concerned with the *volts at terminals*, and not with the combination of windings which produces this voltage.

So far as the wave shape of the terminal voltage is concerned, it may be pointed out that this is not necessarily the same as the wave shape of the E.M.F. developed in the armature windings. Thus, what is known as the third harmonic, and all multiples of the third harmonic, are absent from the voltage measured across the terminals of a star-connected three-phase generator. By the third harmonic is meant a sine wave of three times the periodicity of the fundamental sine wave, which, when superimposed on this fundamental wave, produces distortion of the wave shape.

A voltmeter placed across the terminals of a star-connected generator measures the *sum* of two vector quantities with a phase difference of 60 degrees (see Fig. 33, p. 85). Now, a phase displacement of 60 degrees of the fundamental wave is equivalent to a phase displacement of $60 \times 3 = 180$ degrees of the third harmonic; that is to say, the third harmonics cancel out so far as their effect on the terminal voltage is concerned. The general rule is that the nth harmonic and its multiples cannot appear in the terminal voltage of a star-connected polyphase generator of n phases. The same arguments apply to the line *current* of a *mesh*-connected polyphase generator: the nth harmonic of the current wave can circulate only in the armature windings; it cannot make its appearance in the current leaving the terminals of the machine.

Two-Phase Armature Connections.—It is generally advis-

able to keep the two circuits of a two-phase supply entirely separate, in which case the generator would be provided with four terminals and there would be four conductors connecting the machine to the load; each of the two circuits can be treated as a single-phase alternating-current supply, and loaded independently of the other; at the same time, if the two circuits are brought to an induction motor with stator coils correctly spaced, a revolving field will be produced.

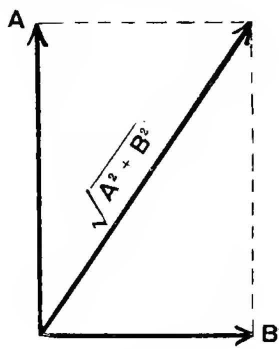

FIG. 35.

Except when transmitting the current to long distances (in which case certain complications arise), a saving of copper may be effected by having one common return conductor which will carry a total current equal to the vector sum of the two outgoing currents.

If the amounts of the two outgoing currents are respectively A and B, the return current will not be $A + B$, because the two component currents are not in phase; it will be equal to $\sqrt{A^2 + B^2}$ for a phase difference of 90 degrees, as will be evident from a glance at Fig. 35.

Thus, if—as would be the case on a load consisting of induction motors—the currents are equal in the two phases, the return current, in the common conductor, will be only $\sqrt{2}$ times as great as the current in any one phase.

The equivalent to the delta or mesh connection of the three-phase generator would be obtained by taking four tappings off the closed armature winding, instead of three as in Fig. 31. A generator with windings connected in this manner may, therefore, be considered indifferently as a two-phase or a four-phase machine.

35. **Regulation of Synchronous Generators.**—The polyphase generators we have been considering belong to the class known as synchronous machines, to distinguish them from the asynchronous generators which are practically polyphase induction motors reversed—*i.e.*, of which the rotor is mechanically driven. This latter type of machine will be briefly referred to after the induction motor has been more fully considered.

Since the synchronous polyphase generator is very similar to its prototype, the single-phase alternator, the scope of this book will only permit of the following points being treated somewhat superficially. At the same time, the manner in which the regulation of an alternator depends upon the nature of the external load is sufficiently important to claim our attention.

In a continuous-current dynamo, the drop in pressure, at constant speed, between no load and full load, is due not only to the resistance of the armature windings, but also to the armature reaction—*i.e.*, to the distorting and demagnetising effects of the armature current. The same thing occurs in a polyphase generator, and, if the load is non-inductive—that is to say, if the current is in phase with the potential difference at generator terminals

—the machine will behave much as a D.C. dynamo in respect to the question of voltage drop.

In Fig. 36, the alternate poles are supposed to be travelling at the back of the armature coils, from right to left. The rectangles A, B, and C represent three different instantaneous positions of an armature coil relatively to the poles. In position A the induced E.M.F. is at its maximum, and tends to send a current in the direction of the small arrows. In position B the E.M.F. is zero (since there is no change in the number of magnetic lines passing through the coil), while in

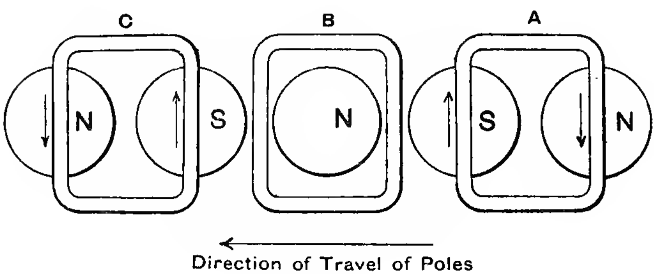

FIG. 36.

position C the E.M.F. is again at its maximum, but in the opposite direction to what it was in position A.

If the load is non-inductive, the current will be in phase with the E.M.F., and, although it will tend to *distort* the magnetic field, it will have no direct effect in either strengthening or weakening the inducing field as a whole. Now imagine the load to be inductive; the current will lag behind the E.M.F., and will, therefore, reach its maximum value when the pole-pieces have travelled beyond the position indicated at A. The result

will be that the current in the armature coils will have a demagnetising effect on the field magnets. We have only to imagine the worst possible condition, of the current lagging 90 degrees, to see that the poles will occupy the position B relatively to the armature coil *when the current in the latter is at its maximum*, and the effect of such a current will evidently be to *weaken* the pole which is opposite the coil, thus diminishing the total flux and causing a drop of pressure at the terminals.

If the current is in *advance* of the E.M.F.—which may occur on a circuit of large capacity—the effects would be exactly opposite to those obtained with a lagging current: the poles would be *strengthened*, and this strengthening effect might even be sufficient to counteract the weakening due to distortion,* and the drop due to armature resistance and reactance; the result being that the peculiar nature of the load might alone be sufficient to maintain constant volts at the terminals of the machine.

36. **Compounding Synchronous Generators.**—The foregoing remarks will make it clear that, in order successfully to compound an alternating-current generator, the increased field excitation must not merely depend upon the amount of the armature ampere turns, but also upon the nature of the load—*i.e.*, on the power factor—and whether the armature current is leading or lagging. In practice the necessity of dealing with the question of heavy leading currents will not arise, although such currents will frequently occur, at *light loads*, on a circuit having large capacity, such as would be obtained with an extensive system of underground cables. The requirements of a perfect alternator, to give constant volts under

* Which, by crowding the magnetic lines to one side of the pole-pieces, increases the magnetic reluctance, and diminishes the total flux through the armature.

all conditions of load, when driven at constant speed, may be summed up as follows:

(1) The terminal pressure should remain constant notwithstanding alterations in amount of the armature current or in the power factor (within practical limits).

(2) The compensating windings or devices should be such as to be practically instantaneous in their operation, so that the generator may respond promptly to the altered conditions of load.

(3) The compensating or compound devices must not interfere with the successful running of the machines in parallel.

This last condition may be disposed of at once by stating that, in the case of compounded alternators, it may be just as necessary to provide an equalising bar and equalising connections between the machines as when running direct-current compound-wound dynamos in parallel.

With regard to the condition (2), this would appear to remove from the field of useful competition all automatic devices, the principle of which depends upon the variation of the field current by the cutting in or out of resistances. It should be pointed out, in this connection, that, even if the current round the field magnets were to alter instantly with the change of load, it might yet be a matter of several seconds before the terminal pressure should regain its correct value owing to the large inductance of the field circuit.*

The nearest approach to perfect regulation in alternators by varying the resistance in series with the field coils is to be found in the Tirrell regulator, which is much used on the continent of America. Its success is due largely to care in design and attention to minor

* See article 13, Chapter II., p. 28.

details, such as the elimination of destructive sparking at the relay contacts. The main principle is simple, and well worth a brief description even in these pages, from which detailed description of commercial apparatus and auxiliary devices are of necessity excluded. By means of certain magnets and solenoids, any fall of exciter voltage, or terminal voltage of the generator, automatically short-circuits a very large section of the field rheostat. The greatly increased field current raises the alternator voltage, with the result that the short circuit on the rheostat is again automatically removed. The operation is repeated at a rapid rate, and the results obtained are hardly comparable with the necessarily sluggish regulation obtained by cutting in or out segments of the field regulating switches in the ordinary way, whether by hand or by automatically controlled power.

Probably the most successful compound generator on the market is the self-exciting synchronous machine devised by Mr. Alexander Heyland, whose work in connection with induction motors and asynchronous generators is well known. It is not proposed to describe his method in these pages, as this cannot be done in a few words; but it may be stated that, not only the *amount* of the armature current, but also the *power factor* is taken into account, and actual tests have shown that, even on zero power factor, the pressure at terminals will remain almost constant for any current up to the full-load current taken from the machine.*

* The reader who cares to go further into this matter is referred to Mr. Eborall's articles in the *Electrician* of July 3 and 10, 1903, and July 15, 1904; also to the excellent paper on compensated alternate-current generators read by Mr. Miles Walker before the Manchester Section of the Institution of Electrical Engineers, November 29, 1904.

There would appear to be a large field for the ingenuity of inventors in this matter of self-compounding alternators, but it is questionable whether the demand for such machines justifies the increase of cost and possible unreliability due to any departure from the simplicity of the ordinary designs. An automatic regulator external to the machine, such as the one above described, appears to fulfil the requirements satisfactorily, notwithstanding the time lag that must necessarily accompany any change of current in the field windings.

37. **Parallel Running of Alternators.** — An attempt will be made briefly to state the reasons which account for the successful parallel running of machines of the type we have been considering; but it is not intended to investigate in detail the actions that take place under various conditions of driving or of load.

In the first place, it must be recognised that the synchronous alternator is a reversible machine—that is to say, it will run as a motor when supplied with power from an alternating source. The reasons which account for the continuous-current dynamo being a reversible machine should be well known to every reader of these pages, and it should, therefore, be merely necessary to point out that when an alternator is used as a motor, exactly the same forces come into play between armature conductors and field, *provided the armature is supplied with current at the correct frequency, and that the revolving portion of the machine has been run up to the correct speed, known as the speed of synchronism.*

Imagine an alternator running as a generator on load at a definite speed, or frequency. The torque exerted by the prime mover is required—as in the case of a dynamo —to force the conductors carrying current across the

magnetic field.* If, now, we suppose the prime mover to be disconnected, and, before the generator has had time to slow down, we connect the armature terminals to an alternating-current supply, the machine will continue to run as a motor, provided the frequency of the supply is correct—*i.e.*, such as to correspond with the speed and the number of poles.

The current will flow in the armature coils in the reverse direction, for corresponding positions of armature coils and poles, the result being a number of impulses tending to keep the machine *in step*. If load is thrown on the machine, there will be a momentary retardation, causing a slight displacement of phase between the back E.M.F., due to the conductors cutting the magnetic flux, and the applied potential difference: these forces will not be exactly opposed to each other, and the result will be an increase of current which will be sufficient to keep the machines in step. If the machine breaks out of step, which might occur, for instance, on a sudden and considerable overload, then, since its speed is no longer that of synchronism, the regular succession of impulses is replaced by a number of irregular impulses in both directions, and the machine will quickly come to rest.

Although the synchronous alternating-current motor is an excellent machine for certain purposes, it has the disadvantage of not being self-starting.† When running, it

* It is true that the forces exerted in the case of a single-phase alternator are intermittent, being of the nature of a series of impulses, and not continuous as in a dynamo or a polyphase machine; but this does not alter the argument.

† The synchronous polyphase motor will sometimes be self-starting on light loads owing to the rotating magnetic field of the stator currents setting up eddy currents through closed paths in the pole faces of the field magnets, even if no copper coils are specially

has one definite speed, depending upon the frequency of the supply circuit.

With regard to the parallel running of synchronous generators, so far as general principles are concerned, there is little to add to the foregoing remarks. If two or more machines are electrically coupled to common 'bus bars, and if any one of them—owing to a momentary slowing of its prime mover, or from any other cause—lags, only for an instant, behind the others, there will be an immediate rush of current from the 'bus bars, which will tend to pull it into step. Successful parallel running depends less upon the design of the alternators than of the engines; an even turning moment is essential. Large and small units can be run in parallel with the greatest ease; and, although it is desirable that all the machines should give approximately the same shape of E.M.F. wave, this is not essential. The portion of the total load taken up by each of several generators in parallel will depend simply upon the amount of power developed by the respective prime movers. There is also a correct value of the exciting current for each of the several units corresponding to a given load; otherwise the necessary magnetic flux will have to be produced by internal circulating armature currents provided by the other machines. It is true that, before switching an additional machine on to the 'bus bars, it has to be run up to the speed of synchronism; but, once switched in, it is magnetically locked with the

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provided in the pole shoes for this purpose. If this method is adopted, the volts impressed on the stator windings should be reduced below normal, and connections between the (unexcited) field coils should be broken during the period of starting, otherwise there is danger of the total E.M.F. induced in the field circuit being excessive.

other machines, and to those who have had no experience of alternators running in parallel, it is surprising to note how firmly each unit appears to be gripped by the other machines and held in step.

38. **Synchronisers.**—When a synchronous motor is run up to speed by means of an auxiliary induction type motor, or when an extra generator is about to be thrown in parallel with other generators feeding common 'bus bars, it is necessary to know that the speed of synchronism has been reached before closing the main switches. The simplest form of synchroniser is an incandescent lamp connected across the switch terminals. The arrangement for three-phase machines is shown in Fig. 37. As the motor is run up to speed, with its field excited, the lamps will flicker, being brightest at the instant when the phases are so related that the maximum amount of current will tend to pass between the machines. As the speed of the motor approaches synchronism, the "beats" of the lamps will become slower and slower, until two or three seconds can be counted between the successive periods of brightness and darkness. If the switches are then closed at an instant when the lamps are dark, there will be no abnormal rush of current between the machines, because they will be in synchronism. The motor must be provided with a voltmeter so that the field current can be adjusted to give approximately the same voltage as the generator.

The above explains the *principle* of the synchroniser. There are many modified and convenient devices on the market which enable the operator to close the switches at the right instant. It is obvious that a voltmeter or its equivalent can be used in place of the lamp; and in any case, for the higher voltages, the synchronising device, of whatever type, would ordinarily be connected

in the secondary circuit of a step-down transformer. In the arrangement referred to—of three lamps connected directly between terminals—it is not necessary to join these up in the manner indicated in Fig. 37. They can be so connected that the correct instant for closing the switches is when all three lamps are *bright*; or, if desired, an unsymmetrical arrangement of connections can be adopted by which the lamps will glow in succession. This has the advantage of indicating by the *order in which*

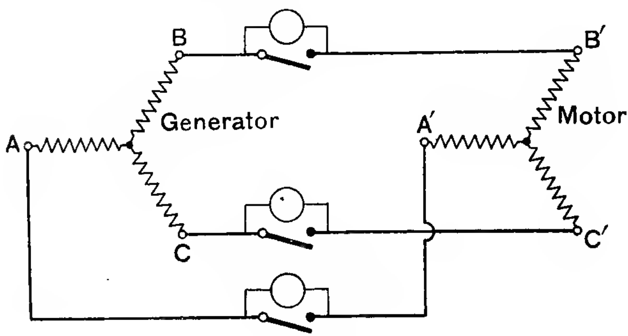


FIG. 37.

the lamps light up whether the incoming machine is running too fast or too slow. If the reader is interested in such details, he should be able to follow out the necessary changes in the lamp connections himself.

Although the incoming machine has been referred to as a motor, the procedure is obviously the same when paralleling generators to meet the demand for additional machines as the load increases.

39. **Output of Polyphase Generators.**—A generator wound to give two or more currents differing in phase will have a larger output than if it is wound to give only

single-phase currents. It is not proposed to study and compare the various methods of winding armatures, in order to show that the polyphase generator has a greater output than the single-phase machine; but, owing mainly to the fact that the whole of the armature surface can be covered by the windings, it is found in practice that, for the same type and dimensions, a machine provided with

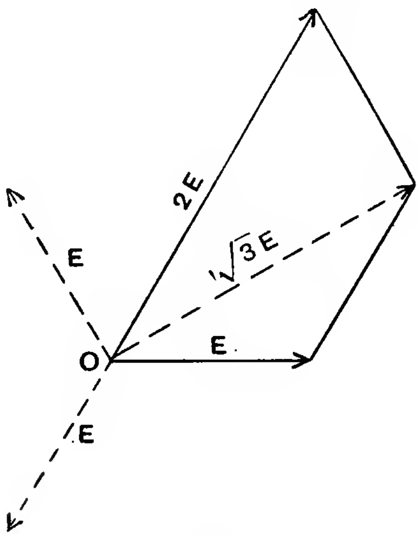


FIG. 38.

windings for two-phase currents will have an output about 30 per cent. greater than if wound for single-phase currents.

If single-phase currents are required, they can always be obtained from polyphase machines. Thus, a two-phase generator would work as a single-phase machine with its two windings coupled in series, the terminal

pressure being $\sqrt{2}$ times the pressure per phase for the same speed and excitation. The output, however, would not be so great, because, although the current, for the same heating effects, would be the same as before, the terminal pressure is not doubled by connecting in series the two (out of phase) windings.

The power of the two-phase machine $= 2\,(E \times I)$, where E and I stand respectively for the pressure and current per phase, the load being assumed non-inductive. The power of the same machine used as a single-phase alternator will be only $\sqrt{2}\,E \times I$ for the same losses in the armature coils, or about 30 per cent. less.

In the case of a three-phase generator, we may connect all three windings in series in the manner indicated by the vector diagram Fig. 38; and, if the E.M.F. in each section is equal to E volts, the resultant E.M.F. will be 2 E. The output, for a current I in the armature conductors, will be $2\,E \times I$ instead of $3\,(E \times I)$, which would be the output for the same heat dissipation in armature coils if the machine were connected up for three-phase working.

A slightly greater output as a single-phase machine will be obtained by utilising only two of the three sections. The resultant E.M.F. will be only $\sqrt{3}\,E$, as shown by the dotted vector in Fig. 38; but for equal $I^2\,R$ losses in armature windings the current may now be

$$I \times \frac{\sqrt{3}}{\sqrt{2}},$$

making *output* $= \dfrac{\sqrt{3}\,E \times \sqrt{3}\,I}{\sqrt{2}}$

$$= \frac{3}{\sqrt{2}}\,E\,I$$

$$= 2{\cdot}12\,E\,I.$$

CHAPTER IV

MEASUREMENT AND CALCULATION OF POWER ON POLYPHASE CIRCUITS

40. It is not proposed to describe in detail the principles of construction of the various types of wattmeters used for measuring power on alternating-current circuits. The single-phase wattmeter may be considered as a modified form of Siemens dynamometer, with fixed thick-wire coil and movable fine-wire coil. The fixed coil carries the main current, while the movable coil is connected in series with a large non-inductive resistance, and takes a current proportional to the potential difference. The object of the non-inductive resistance is to reduce the "time constant"* of the shunt circuit to a negligible

* It was explained in article 13 how the *time* required for the growth or decrease of the current depends upon the self-induction of the circuit; but the effects of self-induction will depend very much on the *resistance* of the circuit. If the resistance is only high enough, the counter E.M.F. of self-induction will become a negligible quantity in comparison with the E.M.F. required to overcome the resistance, and the *time constant* of a circuit may be defined as the ratio of its coefficient of self-induction to its resistance, or

$$\textit{time constant} = \frac{L}{R}.$$

It follows that, in a circuit of which the time constant is small, the effects of self-induction may be negligible whatever may be the actual value of the E.M.F. induced by the passage of unit current.

amount, in order that the current in this circuit may always be in phase with the potential difference across the terminals.

If I is the current in the fixed coil, and I_E the current in the movable coil, the reading of the wattmeter will be proportional to $I\ I_E \cos\theta$, where θ is the phase angle between the two currents; and, since I_E is strictly proportional to the terminal potential difference, the instrument will indicate the true power of a single-phase alternating-current circuit.

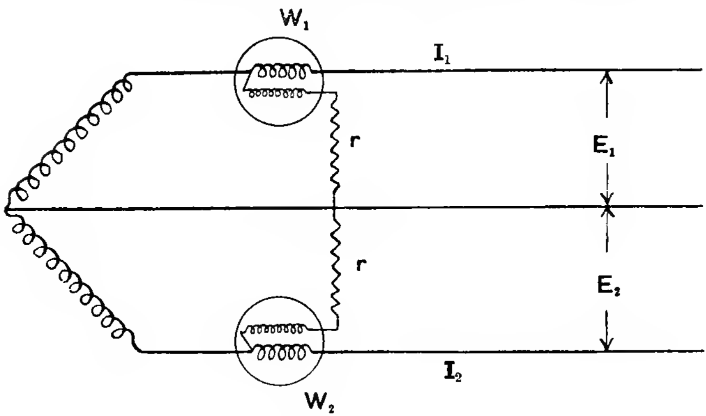


FIG. 39.

It should be mentioned that, even when the wave shapes are irregular and dissimilar—in which case the phase angle θ must be considered as the displacement between *equivalent* sine waves—the wattmeter will still correctly indicate the true power in the circuit. The force tending to produce deflection of the pointer is at every instant proportional to the product of the currents

in the two coils; and since the inertia of the moving parts does not permit of the pointer following the changes of power which occur throughout a complete cycle, the steady deflection obtained is actually an indication of the *average* value of the power in the circuit. Thus, even in the case of irregular-shaped waves as in Fig. 10, p. 22,

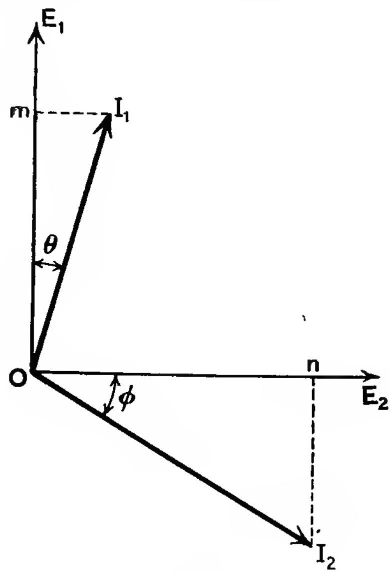


FIG. 40.

the wattmeter measures the average ordinate of the shaded power curve, and automatically subtracts the negative portions of the power from the positive portions.

41. **Power of Two-Phase Circuit.**—If the two circuits are separate—*i.e.*, if the distribution is by means of four wires—two wattmeters would be required—one in

each circuit—and the sum of their readings would give the total power of the two phases.

If the load is balanced, as will be the case if it consists of induction motors only, the pressure, current, and angle of lag in the two phases will be the same, and one wattmeter will suffice; it is merely necessary to double its reading to obtain the total power of the two-phase circuit.

If there is a common return conductor for the two phases, one wattmeter will also suffice, provided the load is balanced. If the distribution of load is unequal, two wattmeters are required, as shown in Fig. 39.

The vector diagram, corresponding to Fig. 39, for a partly inductive unbalanced load has been drawn in Fig. 40.

Here the vectors O E_1 and O E_2, drawn at right angles to each other, and equal in length, represent the pressures E_1 and E_2, with a phase difference of 90 degrees.

The vector O I_1 shows the current I_1 lagging behind its E.M.F. by an angle θ, while O I_2 represents the current in the other phase lagging behind its E.M.F., E_2, by an angle ϕ. The total power is evidently equal to the sum of O $E_1 \times$ O m, or $E_1 I_1 \cos\theta$ (measured by wattmeter W_1), and O $E_2 \times$ O n, or $E_2 I_2 \cos\phi$ (measured by wattmeter W_2).

42. **Power in Three-Phase Circuit.**—In a three-phase system, transmitting power by means of three wires only, any one wire may be considered as the return conductor for the other two, the current in one wire being always equal to the sum of the currents in the other two wires, these being, of course, combined in the usual way, paying due attention to their phase relation.

The total power may, therefore, be measured by means of two wattmeters only, exactly as in the case of a two-phase unequally loaded system. The wattmeters are in-

serted in any two of the three conductors, while the free ends of the pressure coils are connected—through suitable non-inductive resistances—to the third conductor, as shown in Fig. 41. The sum of the two wattmeter readings will correctly indicate the total three-phase power, whether the load be balanced or not. It is evidently of no consequence whether the generator windings, or the load, be star or mesh connected, *provided the distribution is by three wires only;* but if the generator windings are star-connected, and a fourth conductor is run between the neutral point and the load, for the purpose of connecting

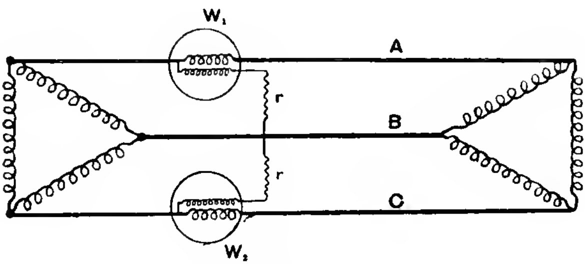


FIG. 41.

lighting circuits between phases and neutral conductor, then three wattmeters will be necessary, as shown in Fig. 42. The free ends of the three shunt windings are connected to the neutral point, or return conductor, D, and the sum of the readings on the three instruments will evidently be the total power on balanced or unbalanced load—*i.e.*, even if there is a current flowing in the common conductor, D.

Balanced Load.—When the load consists of induction motors only, the currents in the three conductors will be equal, and they will also be equally displaced in phase relatively to the corresponding E.M.F.s.

There are two cases to be considered: (1) the case of star-connected generators with neutral point available, and (2) the case of mesh-connected machines in which there is no neutral point available. The arrangement shown in Fig. 42 is applicable to case (1), if we imagine the lamp load between the various phases and the common conductor, D, to be done away with. The load being balanced, the currents in A, B, and C will be equal, and equally displaced in phase relatively to the respective E.M.F.s measured between each of the three terminals and the neutral point: the readings of the three watt-

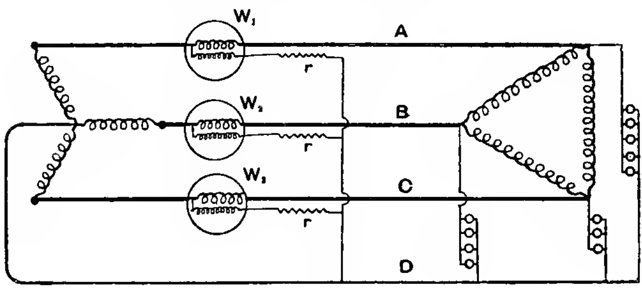


FIG. 42.

meters, W_1, W_2, and W_3, will be identical, and any one of these meters will, therefore, suffice to measure the load on a balanced three-phase circuit; it will be merely necessary to multiply the reading by three.

Consider, now, case (2), in which the neutral point is not available. The arrangement is as shown in Fig. 41, with the exception that one of the wattmeters—say W_2—is supposed to be entirely dispensed with. The remaining wattmeter, W_1, with the free end of the fine-wire coil connected to B (as shown in the figure), will indicate the quantity E I cos (30 degrees − θ), where θ is the angle of

lag (cos θ being the power factor). If, now, we remove the free end of the fine-wire coil from B to C—leaving the series coil in circuit with the conductor A—the instrument will indicate the quantity E I cos (30 degrees + θ). The sum of these two expressions—after the necessary simplifications have been effected—amounts to $\sqrt{3}$ E I cos θ, which is the expression for the total power, where I and E stand respectively for the current in any one conductor and the pressure between phases of a balanced three-phase system. Thus, even when the neutral point is not available, the sum of two readings taken on a single wattmeter will give us the true power of the balanced three-phase circuit.

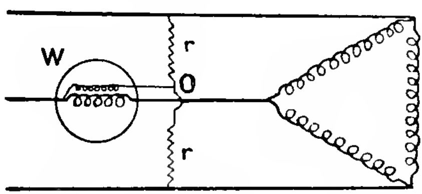


FIG. 43.

The method of taking two readings on a single wattmeter would not be convenient in practice, neither is it necessary. It is possible to combine the shunt currents due to the E.M.F.s between A and B, and A and C, respectively (Fig. 41), so as to obtain a total current in the fine-wire coil which shall give the required reading on the one wattmeter.

Instead of changing over the shunt connection of the single wattmeter from B to C (Fig. 41), it is merely necessary to provide two non-inductive resistances *each* of *r* ohms (as before), and connect them respectively to the two conductors in which there is no wattmeter, as

shown in Fig. 43. The wattmeter will then directly indicate the total power of the balanced circuit.

This arrangement is very similar to an alternative method which consists in producing an *artificial neutral point* by means of a star resistance, or equivalent arrangement in which a choking coil is used: it is, indeed, only necessary to imagine a third resistance, of r ohms, inserted between the fine-wire coil of the wattmeter and the point O. This—on the assumption that the shunt-coil resistance is negligible—would have the effect of reducing the current in the shunt coil to one-third of its previous value, and the wattmeter would, therefore, indicate one-third of the total power, exactly as if it were connected to the neutral point of a star-connected generator supplying a balanced load.

43. **Vector Diagram for Calculating Star Resistances.** — The diagram Fig. 44 will serve to indicate how the required non-inductive resistance may be calculated to give any desired shunt current through the wattmeter or other instrument connected in one phase only.

Let a b c be the E.M.F. triangle—the sides of which are equal, and indicate the pressures between phases—while c C represents the required current in shunt coil.

The first and all-important point to bear in mind in the construction of such diagrams is that the current, C, leaving any one terminal must exactly equal the resultant of the other two currents, A and B; in other words, the sum of the currents at the three terminals is always equal to zero.* The first condition to be fulfilled in Fig. 44 is, therefore, that the three current vectors a A, b B, and c C shall form a closed triangle.

* The only exception being the case of a star-connected generator with a fourth conductor returning to the neutral point.

Draw the dotted line B A parallel to the side *b a* of the pressure triangle, and at a distance below it equal to half the length of the (known) current vector *c* C. If, now, we join any point such as O_1—lying on the prolongation of the vector *c* C, within the triangle *a b c*—to the vertices *a* and *b* of the pressure triangle, it will be seen that the prolongation of the lines until they cross the dotted line

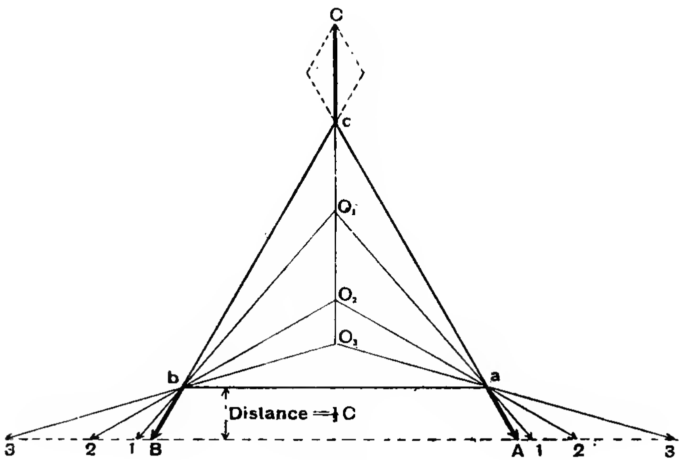


FIG. 44.

B A will give us the current vectors *a* 1 and *b* 1, which, when combined together, will exactly balance *c* C.

It will be seen that there are many solutions of the problem, but the required ohms in the three arms forming the star resistance are readily calculated for any position of the point O.

The drop of pressure in each arm of the non-inductive resistance is represented by the lengths of the vectors O *a*, O *b*, and O *c* (in phase with the corresponding

currents), and the necessary ohms in each arm can readily be calculated by dividing the lengths of these pressure vectors by the lengths of the corresponding current vectors—the scale to be used for the pressure vectors being, of course, the same as used for drawing the triangle *a b c*.*

The total watts lost in the star resistance will be equal to the sum of the $I^2 r$ losses in the various arms of the star. It will be found that, if *c* C is kept constant, these losses increase as the point O is taken lower down. They will be greater for the system of vectors drawn from the point O_3 than for that which has for the common point O_2 (the centre of the triangle); but the most economical arrangement consists in doing away with the resistance in series with the current C. This brings the point O to coincide with *c*, and makes the total power absorbed by the resistances only two-thirds of what it would be if a star resistance with three equal arms were used. It will be observed that the connections in this case would be exactly as shown in Fig. 43.

44. **Power Factor of Three-Phase Circuit.**—The *power factor* of any circuit carrying an alternating current may be defined as the ratio $\dfrac{\textit{true power}}{\textit{apparent power}}$. In the case

* It should be clearly understood that the sides of the triangle *a b c* represent, by their length and direction, the magnitude and phase relations of the pressures *measured between conductors*, while the star O *a*, O *b*, O *c*, represents the equivalent system of vectors all radiating from a common point O. That such a system of star vectors is equivalent to the triangle of vectors is proved by the fact that *a b* is (vectorially) equal to O *b* - O *a*. Similarly of the other vectors:

$$b\,c = O\,c - O\,b$$

and

$$c\,a = O\,a - O\,c.$$

of a single-phase circuit, this expression may be written,

$$\textit{power factor} = \frac{\textit{watts}}{\textit{volt-amperes}};$$

but in polyphase circuits it is not always clear what is to be understood by the total volt-amperes or apparent power. In a balanced three-phase circuit, with the angle of lag, θ, the same on all three phases, the true power is $\sqrt{3}$ E I cos θ, and the expression $\sqrt{3}$ E I, which stands for the total volt-amperes, will be more clearly recognised when put in the form $3\left(\frac{E}{\sqrt{3}} \times I\right)$, where $\frac{E}{\sqrt{3}}$ is the star voltage; the total apparent power, in this case, being the sum of the volt-amperes in each branch of the three-phase supply.

If the expressions for power are divided by the voltage, the power factor can be expressed by the ratio $\frac{\textit{total energy current}}{\textit{total current}}$, and this is, perhaps, the most convenient form of the expression for general use, seeing that the three voltages are usually equal and symmetrical as regards phase angle. In the case of a *balanced* inductive load, the "total" current for use in the power factor formula would be $3 \times I$, and the "total" energy current would be $3 \times I \times \cos\theta$; but when the load is not balanced the expression is not quite so simple. Not only the magnitude of the (different) angles of lag, but also their *sign—i.e.*, whether the current vector leads or lags behind the star vector of the E.M.F.—must be taken into account; this will be explained in article 46.

45. **Vector Diagram for Balanced Inductive Load.**—A condition very frequently met with in practice is that of generators supplying current to induction motors only.

In this case the power factor will be less than unity; but the load will be equal on all three phases.

Such a load is shown diagrammatically in Fig. 45.

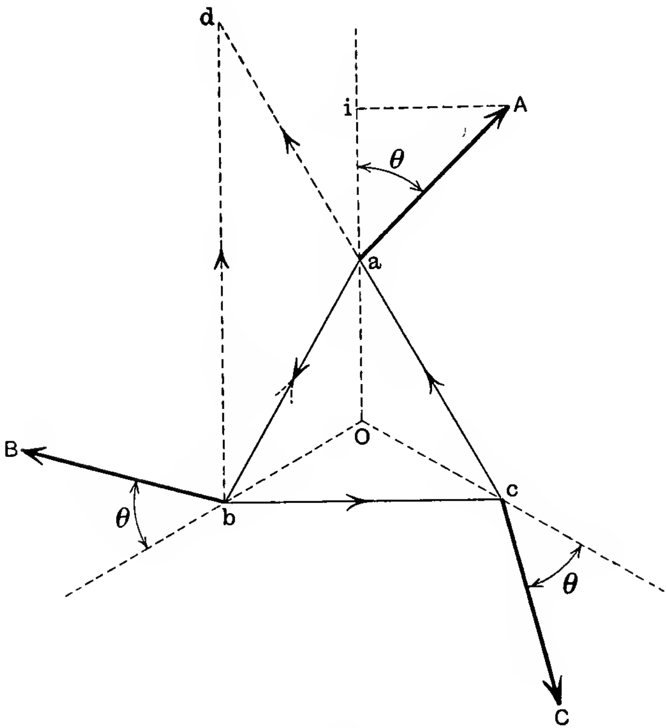


FIG. 45.

Here the current vectors A, B, and C are all equal in length; but, instead of being in phase with the pressure vectors *o a*, *o b*, and *o c*, they lag behind these vectors by a certain angle θ.

The total power is

$$\begin{aligned} W &= 3\,(o\,a \times a\,i) \\ &= 3 \times o\,a \times a\,\mathrm{A} \cos\theta \\ &= \sqrt{3}\,\mathrm{E\,I} \cos\theta \end{aligned}$$

where E is the pressure between phases;
I is the amount of any one of the three line currents; and
cos θ is the power factor of the balanced three-phase circuit.

Measurement of Power Factor on Balanced Load.—There are instruments called phase meters, or power factor indicators, so constructed as to show, by the position of a pointer, the phase angle θ between current and E.M.F.; but, by means of a voltmeter, ammeter, and wattmeter, this angle can evidently be calculated: the two former instruments will enable us to calculate the *apparent* power ($3 \times o\,a \times a\,\mathrm{A}$), while the wattmeter will directly indicate the true power ($3 \times o\,a \times a\,i$), and the ratio which this latter quantity bears to the apparent power will be the *power factor* ($\cos\theta$) of the three-phase circuit.

It is, however, possible to determine the angle θ, even if neither ammeter nor voltmeter is available, by the use of a wattmeter only.

It has already been explained (see article 42, p. 106) how the total power of a balanced three-phase circuit may be measured by means of a single wattmeter connected in series with one main, when the shunt coil is connected alternately to each of the other two mains, the sum of the two readings giving the total power.

These two wattmeter readings will also enable us to

calculate the power factor of the load. Let W_1 and W_2 be the two readings of the wattmeter; then, if θ is the angle of lag, it can be shown that

$$\tan \theta = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}.$$

From this we obtain the angle θ, and $\cos \theta$, the power factor of the balanced circuit.

That the above formula is correct for a power factor of unity is evident, because W_1 will be equal to W_2, thus making the numerator of the right-hand side of the equation equal to zero; and if $\tan \theta = 0$ it follows that $\theta = 0$, which is the condition required to make the power factor ($\cos \theta$) equal to unity. By the aid of Fig. 45 it will be seen that the above formula is correct for any value of the angle θ.

Let us suppose the current A to pass through the series coil of the wattmeter, and the reading W_1 to be taken with the free end of the shunt coil connected to *c*, while for the reading W_2 this connection is transferred to *b*.

The power W_1 will be equal to the product of the volts *c a* by the current *a* A, or to the length *c a* multiplied by the projection of *a* A upon *c a*. Similarly, the power W_2 will be equal to *a b* multiplied by the projection of *a* A on *a b*.

It will be observed that, in the two readings, the *amounts* of the main and shunt currents remain the same, but the *phase* of the shunt current is different. The results obtained by adding or subtracting the two wattmeter readings would evidently correspond with a single reading of the wattmeter obtained by passing through the shunt coil a current equal to the sum or difference—as the case may require—of the two separate

shunt currents. Now, the *sum* of the two vectors $c\ a$ and $a\ b$ is $c\ b$; and their *difference*—which is the same thing as the sum of $c\ a$ with $a\ b$ *reversed*—is $b\ d$, exactly at right angles to $b\ c$, and equal in length to $\sqrt{3}$ times $b\ c$. We may, therefore, write

$$\begin{aligned}(W_1 + W_2) &= b\ d \times \text{the projection of } a\ A \text{ upon } b\ d \\ &= b\ d \times a\ i \\ &= \sqrt{3}\ b\ c \times a\ i \qquad . \quad . \quad . \quad . \quad . \quad (1)\end{aligned}$$

and $$(W_1 - W_2) = b\ c \times i\ A \qquad . \quad . \quad . \quad . \quad (2)$$

Inserting the values (1) and (2) in the original equation, we have

$$\begin{aligned}\tan\theta &= \frac{\sqrt{3} \times b\ c \times i\ A}{\sqrt{3} \times b\ c \times a\ i} \\ &= \frac{i\ A}{a\ i},\end{aligned}$$

which, since $a\ i\ A$ is a right-angled triangle, is the definition of the trigonometrical tangent of the angle θ, and proves the correctness of the formula.

(A little difficulty may be experienced in understanding why the *difference* of the vectors $c\ a$ and $a\ b$ has been taken to obtain the equivalent resultant vector from which the *sum* of the wattmeter readings is calculated, and *vice versa*; but this is due to the method of changing over the shunt connection, leaving the *same* end of the shunt coil permanently connected to the main A in which the wattmeter is placed.)

Example.—Let us suppose that the wattmeter reading $W_1 = 20$ kilowatts and $W_2 = 10$ kilowatts; then

$$\begin{aligned}\tan\theta &= \frac{\sqrt{3}\ (20 - 10)}{20 + 10} \\ &= \cdot 577.\end{aligned}$$

Referring to a table of natural tangents, we see that this corresponds to an angle $\theta = 30$, and since cos 30 degrees $= \cdot 866$, this will be the power factor of the balanced three-phase circuit.

46. **Unbalanced Inductive Load.**—Consider a three-phase system in which a lamp load is taken off one or more phases, in addition to induction motors on all three phases. This may be effected (1) by connecting the lamps in delta fashion directly between the conductors A B, B C, or C A, or (2) if the generator armature windings are star connected, by running a fourth wire, D, back from the load to the neutral point, and connecting the lamps between the terminals A, B, or C and this fourth conductor, as shown in Fig. 42 (p. 108). In either case the three current vectors A, B, and C (Fig. 46) may be unequal in length, and make different phase angles, α, β, and γ, with the equivalent star vectors drawn from the point O (the centre of the triangle); but in case (1) these three vectors will form a closed triangle when combined together, whereas in case (2) the vector representing the current (if any) in the fourth conductor, D, will be required to close the polygon of the current vectors.

The question now arises as to what is to be understood by the power factor of such a load. It may still be defined as the ratio of the real power to the apparent power; but although the real power is readily measured or calculated as already explained, it is not so easy to define what is now to be understood by the apparent power of the unbalanced three-phase circuit.

If, however, we consider the total current as being made up of two factors—the total "energy" or "active" current and the total "wattless" or "reactive" current—we shall, by combining these at 90 degrees, obtain a quantity

which may be used for calculating the power factor in all cases, whether the load be balanced or otherwise.

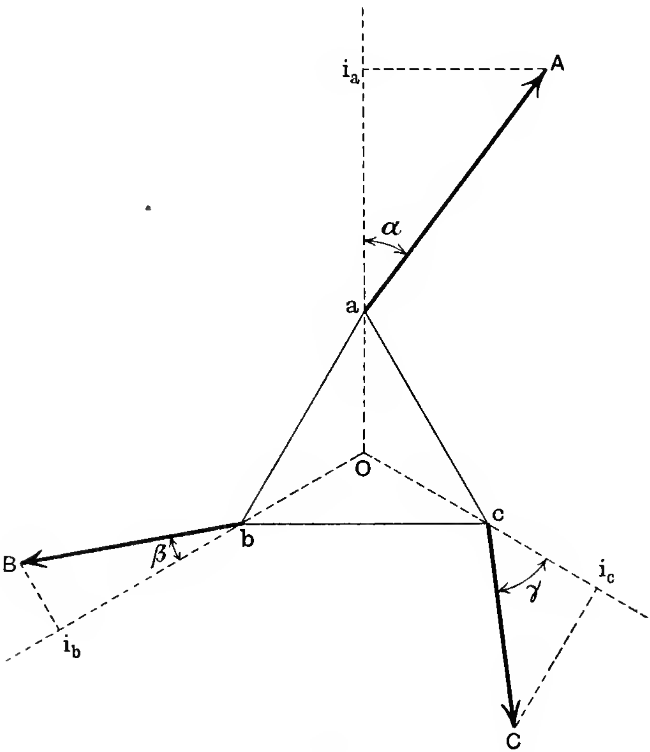


FIG. 46.

Thus, the "total" line current, instead of being written A + B + C, should be expressed as

$$\sqrt{(a + b + c)^2 + (i_a + i_b + i_c)^2},$$

where a, b, and c stand for the "energy" components of the currents—A cos α, B cos β, and C cos γ—and i_a, i_b, and i_c stand for the "reactive" or "wattless" components, A sin α, B sin β, and C sin γ. When the *sign* of this idle current component is taken into account, it will be seen that the above expression may give results differing appreciably from the arithmetical sum of the idle currents in the lines; but, in the event of the load being entirely non-inductive, the algebraic sum of the idle currents will be zero, thus making the apparent power, as calculated by this method, equal to the true power.

The power factor of an unbalanced three-phase load may, therefore, be expressed by the ratio

$$\frac{I_w}{\sqrt{I_w^2 + I_o^2}},$$

where I_w = the sum of all energy components of current,
and I_o = the (algebraic) sum of all wattless components of current.

If we adopt the lettering of Fig. 46, the power factor would be written

$$\frac{(a\,i_a + b\,i_b + c\,i_c)}{\sqrt{(a\,i_a + b\,i_b + c\,i_c)^2 + (A\,i_a + B\,i_b + C\,i_c)^2}}.$$

Example.—Suppose the pressure between terminals to be 500 volts, and let the phase angles be respectively

α = 45 degrees of *lag*,
β = 10 degrees of *lag*,
γ = 15 degrees of *lead*,

and assume the main currents, as indicated by the line ammeters, to be

$$\begin{aligned} A &= 140 \text{ amperes.} \\ B &= 215 \quad \text{,,} \\ C &= 105 \quad \text{,,} \end{aligned}$$

Then
$$\begin{aligned} a\, i_a &= A \cos \alpha = 99 \\ b\, i_b &= B \cos \beta = 211{\cdot}7 \\ c\, i_c &= C \cos \gamma = 101{\cdot}4 \\ & \text{Total} \;\ldots\; 412{\cdot}1 \end{aligned}$$

Also
$$\begin{aligned} A\, i_a &= A \sin \alpha = +\,99 \\ B\, i_b &= B \sin \beta = +\,37{\cdot}3 \\ C\, i_c &= C \sin \gamma = -\,27{\cdot}2 \\ & \text{Total} \;\ldots\; 109{\cdot}1 \end{aligned}$$

The real power is therefore

$$\frac{500}{\sqrt{3}} \times 412{\cdot}1 = 110 \text{ kilowatts,}$$

and the power factor is

$$\frac{412{\cdot}1}{\sqrt{(412{\cdot}1)^2 + (109{\cdot}1)^2}} = {\cdot}967.*$$

* If it is desired to dispense with the process of squaring and extracting the square root, we can use trigonometrical tables thus :

Let $\cos \theta$ stand for the power factor of the three-phase circuit; then, since

$$\text{mean } \sin \theta = \tfrac{1}{3} \times 109{\cdot}1$$

and
$$\text{mean } \cos \theta = \tfrac{1}{3} \times 412{\cdot}1,$$

$$\tan \theta = \frac{109{\cdot}1}{412{\cdot}1} = {\cdot}2647,$$

which corresponds to an angle θ of $14^\circ\ 50'$. By referring to a table of natural cosines we see that $\cos 14^\circ\ 50' = {\cdot}967$, which is the same result as obtained by the more lengthy process of adding the squares and extracting the square root.

In the March number of *Power* (1906) the author described a

47. **Measurement of Power on High-Tension Circuits.**—In all the foregoing diagrams, the instruments—whether wattmeters or ammeters—have been shown as being connected in series with the main leads. This method, however, only applies to circuits of pressures not exceeding, say, 500 volts. For higher pressures, transformers are generally used, in order that low-tension connections only may be brought to the terminals of the instruments. Even on low-tension systems, series

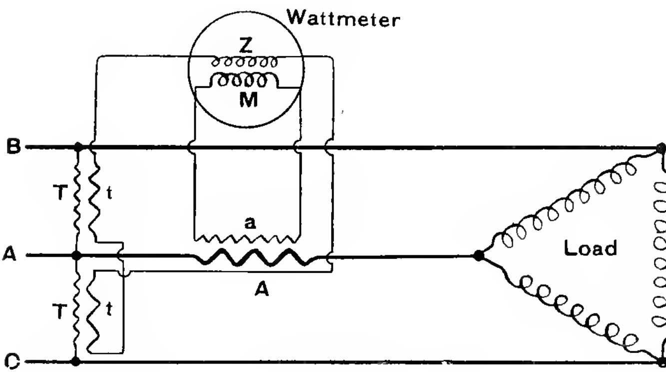


FIG. 47.

transformers would be used to reduce a large current to a convenient amount, such as 3 or 5 amperes, in the instruments. The various methods of power measurements are not altered thereby; it is merely necessary to consider the transformers as suitable pieces of apparatus

simple graphical method which requires only four measurements to be taken off the diagram, and yet gives all particulars for calculating the "real" and the "idle" power, and hence the power factor of any three-phase circuit whether balanced or not.

for providing secondary pressures or currents—as the case may be—*exactly proportional to, and of the same phase as*, the primary pressures or currents.

As an example, the diagram Fig. 47 has been drawn. It illustrates a suitable arrangement for measuring the total power of a *balanced* high-tension three-phase circuit.

Here A and *a* are the primary and secondary wind-

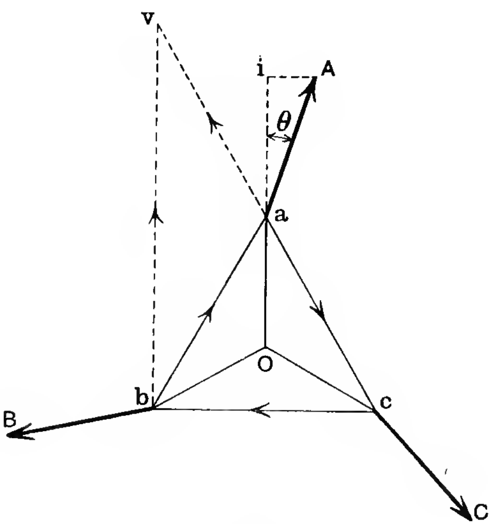


FIG. 48.

ings of the *current transformer* connected in series with one of the mains. This sends a current through the main coil, M, of the wattmeter, of the same phase as the current A, the actual amount of which will depend upon the ratio of the primary and secondary turns, but which —in any case—will be proportional to A.

The two *pressure transformers*, each with T turns in the

primary winding and t turns in the secondary, have their primaries connected up between A and B, and A and C, respectively, while the secondary windings are connected in series, and provide the shunt current for the pressure coil, Z, of the wattmeter.

Fig. 48 will serve to explain how the wattmeter can be made to indicate correctly the total output of the balanced circuit.

It will be seen that the secondary winding of one pressure transformer (Fig. 47) is *reversed* before connecting in series with the other transformer and the pressure coil of the wattmeter. This means that the resultant current through the pressure coil will not be proportional to the *sum* of the vectors $b\ a$ and $a\ c$ (Fig. 48), but to their *difference*, which is represented by the dotted vector $b\ v$. This last is at right angles to $c\ b$, and parallel to O a; and, since the total output is equal to three times O $a \times$ A $\cos\theta$, or to

$$3\ (\mathrm{O}\ a) \times (a\ i),$$

it is evidently only a question of calibration and of the ratios of turns in the various transformer windings, to insure that the single wattmeter shall register the exact total output of the three-phase high-tension balanced circuit.

CHAPTER V

POLYPHASE TRANSFORMERS

48. **Theory of the Single-Phase Transformer.**—A single-phase alternating-current transformer may be considered as consisting of a core of laminated iron upon which are wound two sets of coils, known as the primary and secondary windings respectively.

If an alternating E.M.F. is applied to the terminals of the primary, this will lead to a certain flux of alternating magnetism being set up in the iron core, which, in its turn, will induce a counter E.M.F. of self-induction in the primary winding, the action being that of a *choking coil* (see article 19, Chapter II.). But since the secondary circuit—although not in electrical connection with the primary—is wound on the same iron core, the variations of magnetic flux which induce the back E.M.F. in the primary will, at the same time, generate an E.M.F. in the secondary coils.

The path of the magnetic lines is usually through a closed iron circuit; and, although in practice there is always a certain amount of leakage or stray magnetism which is not enclosed by the secondary windings, the effects of this magnetic leakage are very small in all well-designed transformers, and it will somewhat simplify the diagrams if we neglect this entirely. The assumption is, therefore, that the whole of the magnetism

required to produce the necessary back E.M.F. in the primary coil passes also through the secondary coils—that is to say, the induced E.M.F. *per turn of wire* is supposed to be exactly the same in the secondary as in the primary coil.

Suppose now that the two ends of the primary winding are connected to constant-pressure mains, and that no current is taken from the secondary winding. Under these conditions, the primary circuit acts simply as a choking coil, of which the self-induction is so great, and the ohmic resistance relatively so small, that no current passes, except the very small amount required to magnetise the core. The induced E.M.F. is, therefore, practically equal and opposite to the applied potential difference at primary terminals, and the relation between the magnetic flux in the core and the primary impressed E.M.F. will be given by the equation

$$E_m = \frac{4 \, \mathbf{N} \, S \, n}{10^8},$$

where E_m stands for the *mean* value, in volts, of the primary E.M.F., and n is the frequency.

The number of turns, S, in the primary of a well-designed transformer is always such that the current required to produce the magnetic flux $\mathbf{N}$ is very small; it is generally somewhere between 2 per cent. and 5 per cent. of the full-load current.

Although the rise and fall of the magnetism will be a quarter of a period out of phase with the E.M.F., the open-circuit primary current will not be entirely "wattless," but may be considered as made up of two components — the "wattless" or true magnetising component, in phase with the magnetism, and the "energy" component, due to hysteresis and eddy currents, in phase

with the impressed E.M.F. The reader is, however, referred to article 24 (p. 50), where the question of magnetising current for a circuit containing iron has already been dealt with.

Since the secondary and primary coils are both wound on the same core, it follows that the actual volts induced will be directly proportional to the number of turns of wire in either coil.

Thus, if the primary winding consists of 1,000 turns of wire, all in series, while the secondary has only 50 turns, the ratio of turns is 20 : 1, and this will also be the ratio of primary impressed E.M.F. to secondary induced E.M.F. on the assumption of there being no magnetic leakage and a negligible IR drop in primary.

For convenience in drawing the diagrams, we shall, in all cases, suppose the primary and secondary windings to have the same number of turns; the transforming ratio will therefore be 1 : 1, and the pressure obtained at secondary terminals will be exactly equal, but opposite in phase, to the applied primary pressure.

This last statement will only be strictly correct if we also assume the ohmic resistance of the coils to be negligible; but the pressure drop due to this internal resistance is always small, and rarely exceeds 2 per cent., even in small transformers. It is easily taken into account, if desired; but since the principles of action only will receive our attention, details of design will not be dealt with, and we may, therefore, neglect the ohmic resistance of both primary and secondary coils. Imagine, now, that the secondary circuit of a transformer, with primary on constant-pressure mains, is closed through a resistance; the resulting current will produce a magnetising force in the core. This magnetising force will not produce a change in the magnetism, because it will be instantly

counteracted by a change in the primary current, which will so adjust itself as to maintain the same (or nearly the same) cycle of magnetisation as before—that is to say, the flux will continue to be such as will induce an E.M.F. in the primary windings equal but opposite to the primary impressed potential difference. These two opposing pressures can evidently not be *exactly* equal, or no current would flow in the primary coils; but since, in

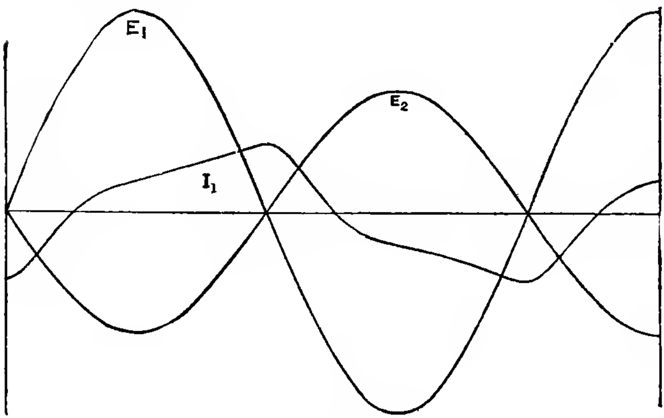


FIG. 49.

practice, the primary resistance—although not of zero value—is relatively small, it will be understood that a very small *resultant* pressure across the primary terminals will cause a very large current to flow through the coils.

In Fig. 49, E_1 is the curve of primary impressed E.M.F., and I_1 is the magnetising current, distorted by the hysteresis of the iron core. E_2 is the curve of secondary E.M.F., which coincides in phase with the

primary induced E.M.F., and is therefore—on account of the comparatively small ohmic resistance of the primary—almost exactly in opposition to the impressed E.M.F. The curve of magnetisation (not shown) would be exactly a quarter period in advance of the secondary E.M.F.

In Fig. 50 the secondary circuit is closed through its proper load of incandescent lamps. There is no appreci-

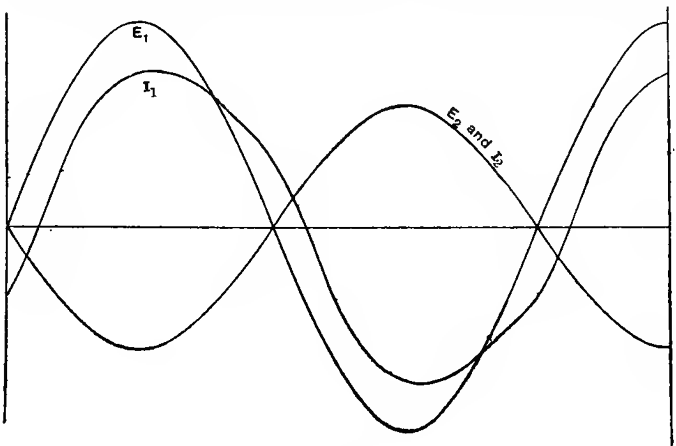


FIG. 50.

able self-induction in such a circuit, and the secondary current will, therefore, be in step with the secondary E.M.F. For simplicity it is represented in Fig. 50 by the same curve as E_2.

The tendency of this secondary current being to weaken the magnetism in the core, and therefore diminish the primary induced E.M.F., it follows that the current in the primary will grow until the magnetism is again of such an amount as to restore balance in the primary

circuit. Hence, neglecting the small loss of volts due to the increased primary current, the induction through the primary must remain as before; and the new current curve, I_1 (Fig. 50), is obtained by adding the ordinates of the current curve in Fig. 49 to those of another curve, exactly opposite in phase to the secondary current, and of such a value as to produce an equal magnetising effect.

49. **Vector Diagram of Transformer without Leakage.**—In Fig. 51 let O E_2 represent the secondary

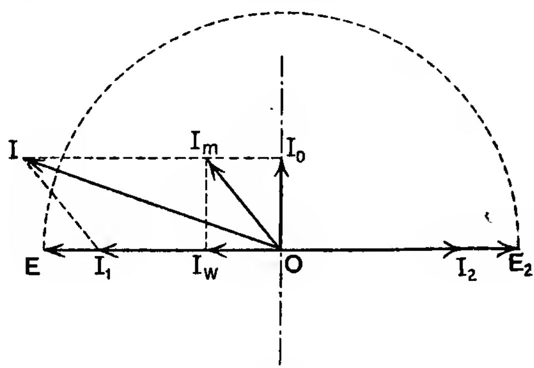


FIG. 51.

E.M.F. Then, on the assumption that the voltage drop in primary, due to ohmic resistance, is negligible—which assumption is generally permissible—the primary impressed E.M.F., E, will be exactly opposite in phase to the secondary or induced E.M.F., and O E will be equal to O E_2 multiplied by the ratio of turns in the two windings. But, since we have assumed—for uniformity and convenience of construction—that the transforming ratio is 1 : 1, the length O E must be made equal to O E_2.

With regard to the magnetising current in the primary coil, which will be very small, this will consist of the "wattless" or true exciting current O I_o, in phase with the induction—and, therefore, 90 degrees in *advance* of O E_2—and the "active" component, O I_w, in phase with O E. This "active" component is due partly to hysteresis and partly to eddy currents, as was explained in Chapter II., article 24, and illustrated by Fig. 20.

The total magnetising current, O I_m, may now be drawn; it will lag behind the impressed E.M.F. by

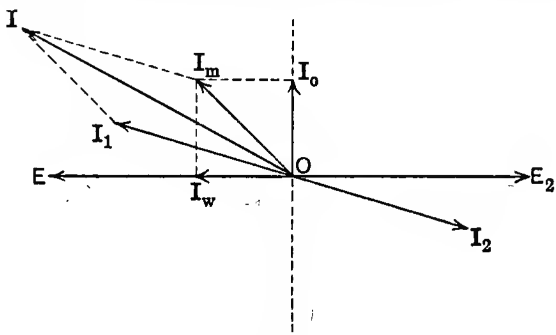


FIG. 52.

about 45 degrees in a well-designed transformer, which corresponds to a power factor, on open secondary circuit, of about ·7.

Effect of Closing Secondary on Non-Inductive Load.—The load being non-inductive, the secondary current, O I_2 (Fig. 51), will be in phase with the secondary E.M.F. It will be balanced by a component, O I_1, of the primary current, exactly equal and opposite to O I_2 (and therefore in phase with E); and the total primary current will now be represented by the resultant O I.

Effect of Closing Secondary on Partly Inductive Load.—In Fig. 52 let O E_2 be the secondary E.M.F. as before, and O I_2 the secondary current, which now lags somewhat behind this E.M.F. The balancing component of the primary current will still be equal and opposite to O I_2, with the result that the primary current, O I, will also lag behind the impressed E.M.F. It will be evident from inspection of the diagram that the energy put into the primary is still in excess of the energy taken out at the secondary terminals by the amount lost in hysteresis and eddy currents in the core.

50. **Polyphase Transformers.**—On any polyphase system, it is always possible to use two or more single-phase transformers, suitably connected up, for the purpose of raising or lowering the pressure of the supply leads.

The use of single-phase transformers has much to recommend it, as, in the event of a breakdown, the repairs are generally more quickly and more economically carried out.

Indeed, for large outputs, it is customary to use separate single-phase transformers on the various phases. A polyphase transformer, built up with a common iron core, might become very large; thus, on a three-phase circuit, it would be required to deal with three times the output of each single-phase transformer, and, apart from the difficulties of manufacture and handling, there would be more difficulty in providing adequate cooling surface, with the result that the saving in cost of material required would be very small.

In the case of transformers for small outputs it is, however, cheaper to arrange the windings on a common iron core. Figs. 53 and 54 show sections through a two-phase and three-phase transformer respectively.

In the former, the primary and secondary coils belong-

ing to phase A are wound on the left-hand limb of the closed iron circuit, while both coils of phase B are wound on the right-hand limb; these two limbs being of equal section. In the centre, between the two bobbins or sets of coils, a common path for the return magnetism is provided, and since this will have to carry either the sum or the difference of the two equal alternating fields having a phase difference of 90 degrees, the centre core—if designed for the same flux density as the two outer cores—must have a cross-section $\sqrt{2}$ times as great as either A or B.

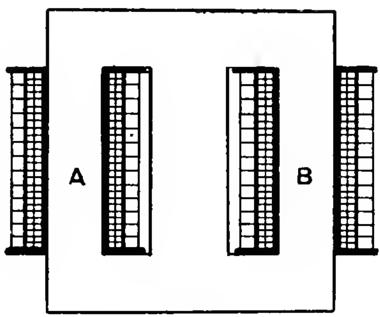


FIG. 53.

In Fig. 54 the three iron cores are of equal section, and they are each wound with the primary and secondary coils belonging to one of the three phases. With this arrangement in place of three separate transformers, if the flux density is supposed to be the same in each case, the saving of iron in the magnetic circuit will be seen to be quite appreciable: it will be noticed that each of the three iron cores forms the return path for the magnetic flux in the other two, in the same manner as the three conductors of a balanced three-phase circuit suffice to

carry the current to and from the apparatus constituting the load.

51. **Methods of Connecting Three-Phase Transformers.**—If it is desirable or necessary to have the neutral point available, the secondary windings of the three-phase transformers must be Y-connected; and for small transformers—in which the space occupied by insulation is large in proportion to the cross-section of the copper in the windings—a saving in cost and weight is effected by adopting this mode of connection.

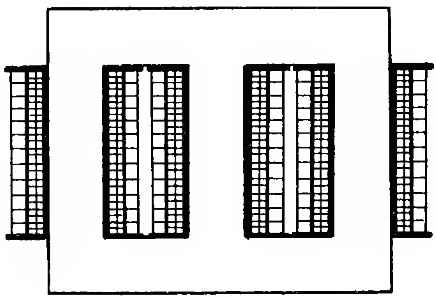


FIG. 54.

The chief advantage of connecting up the windings in Δ fashion, more especially when three separate transformers are used, is that, in the event of one of the transformers in a group of three breaking down, it can be entirely cut out of circuit, leaving the two remaining sound transformers to provide the necessary three-phase pressure; but this will not be possible if the windings are star-connected, since, with this arrangement, any two windings taken together will only provide the pressure across one phase.

With the Δ connection, the removal of one side of the

mesh virtually converts the arrangement into a two-phase system having a common return carrying the same amount of current as each of the other two conductors, instead of carrying a current $\sqrt{2}$ times greater, as would be the case with the more usual two-phase system in which the phase angle is 90 degrees. This is

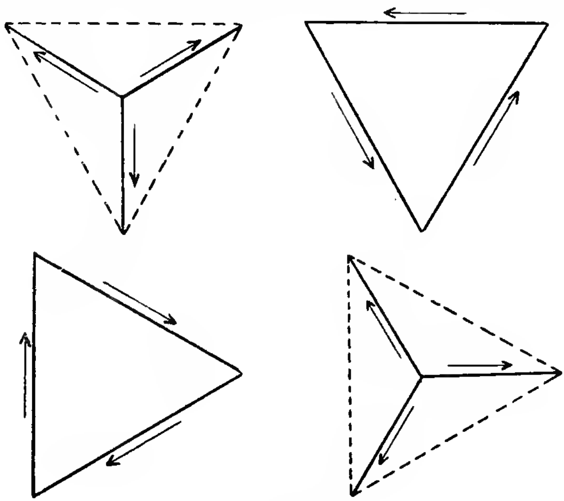


FIG. 55.

known as the open delta connection; but it is used only in cases of emergency.

It is not necessary that the primary windings be connected up in a similar manner to the secondary windings, but if the primary is star-connected and the secondary Δ-connected, or *vice versa*, the ratios of the number of turns in the primary and secondary coils will no longer correspond with the transforming ratio as measured across the phases. The diagrams in Fig. 55 will make this

clear. Here the upper system of vectors indicates the amount and direction of the pressures in the primary coils, while the lower set of vectors indicates the pressures on the secondary side.

The left-hand combination refers to a star-connected primary and a delta-connected secondary, the ratio of transformation being $\frac{1}{\sqrt{3}}$, or ·577 times the ratio of turns; while in the right-hand combination we have a Δ-connected primary and a Y-connected secondary; the transforming ratio in this case being $\sqrt{3}$, or 1·732 times the ratio of secondary to primary turns.

It is interesting to note that, in both these combinations of windings, the secondary pressures *as measured between the three terminals* are not 180 degrees out of phase with the primary pressures (as would be the case if the windings were connected up similarly on both high-tension and low-tension sides), but differ in phase by a right angle, or 90 degrees.

52. **Efficiency of Transformers.**—The efficiency of the alternating-current transformer is very high; its actual value will depend, to a certain extent, upon the skill and knowledge of the designer, but more especially upon the amount and quality of the materials used in its construction.

The efficiency should never be considered apart from first cost, or without reference to the efficiency as a whole of the system in connection with which the transformers are to be used.

It must also be borne in mind that the temperature rise is no indication of the amount of power lost in the transformer; a transformer which gets very hot may be more efficient than another which remains comparatively

cool. Temperature rise must only be considered with reference to the effect it may have upon the materials used in the construction of the transformer; it is a question which concerns the manufacturer.

The efficiency of a small transformer will necessarily be lower than that of a larger transformer. The maximum efficiency will generally be reached at from three-quarters to full load, and it is important that all transformers on a given system be arranged so that the hours during which they supply a *light* load may be as few as possible.

As approximate figures for well-designed transformers, whether single- or poly-phase, ·95 to ·96 may be taken as the full-load efficiency of a 3-kw. transformer, while if the output is one hundred times as great (say 300 kw.), the efficiency might be as high as 98·5 per cent.

53. **Phase Transformation.**—In the last chapter reference was made to the principle of replacing the usual three-phase triangle of pressure vectors by any three *star* vectors radiating from a common point O, and terminating at the vertices of the triangle.

Such a system of vectors will produce exactly the same potential differences between the three terminals as the original mesh connection which it is designed to replace.

Scott's System.—The above principle has been made use of by Mr. C. F. Scott in his ingenious arrangement for changing from two to three phases by means of static transformers only.

In Fig. 56 the left-hand diagram shows the usual three-phase triangle of vectors, which may be replaced by the three vectors O A, O B, and O C, all radiating from the common point O. This point lies on the centre

of the line A C, and the line O B is, therefore, at right angles to A C. In this manner the three vectors A B, B C, and C A can be replaced by two vectors O A and O C exactly equal and opposite, and a third vector O B having a phase difference of 90 degrees with either of the other two.

The right-hand diagram in Fig. 56 shows the transformer connections.

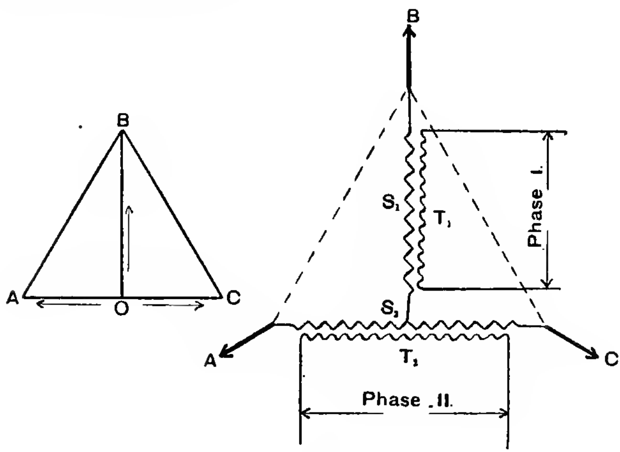


FIG. 56.

Two transformers are needed, with a connection from the end of one secondary to the centre point of the other secondary.

If the primaries of the transformers, T_1 and T_2, are fed respectively by phase I. and phase II. of a two-phase supply having a phase angle of 90 degrees, the pressures generated in the transformer secondaries, S_1 and S_2, will also be 90 degrees out of phase. It is, therefore, merely

necessary so to proportion the number of turns in the windings that the combination of these E.M.F.s will produce equal pressures between the pairs of terminals A B, B C, and C A. If the transforming ratio of transformer T_2 is 1 : 1, the pressure measured across A C will be the same as the two-phase supply pressure; and if the ratio of turns in transformer T_1 were also 1 : 1, the pressure O B (see left-hand diagram, Fig. 56) would be equal in amount to A C. This would not be correct: the length of the vector O B is

$$\begin{aligned} O\,B &= B\,C \sin 60 \text{ degrees} \\ &= A\,C \sin 60 \text{ degrees} \\ &= A\,C \times \cdot 866, \end{aligned}$$

and it follows that, if the ratio of primary to secondary turns in T_2 is 1 : 1, the ratio of turns in T_1 must be 1 : ·866 in order that the triangle A B C may be equilateral.

Example.—Let us assume the two-phase pressure to be 2,000 volts, which it is desired to transform into three-phase at a pressure of 30,000 volts.

Suppose the primary turns on transformers T_1 and T_2 are 200 in each case; then, on *each half* of the secondary winding S_2, the required number of turns is

$$\begin{aligned} \frac{S_2}{2} &= \frac{1}{2} \times \frac{200 \times 30{,}000}{2{,}000} \\ &= 1{,}500; \end{aligned}$$

while the secondary turns on T_1 must be

$$\begin{aligned} S_1 &= \frac{200 \times 30{,}000}{2{,}000} \times \cdot 866 \\ &= 2{,}600. \end{aligned}$$

Lunt's System.—The system of phase transformation described above is reversible—that is to say, it can be used, if desired, for changing from three to two phase.

Another method of effecting this latter transformation is due to Mr. A. D. Lunt. The connections of the two transformers are shown in the left-hand diagram of Fig. 57, while the right-hand diagram shows the vectors of the various *magnetic fluxes* in the cores of the transformers.

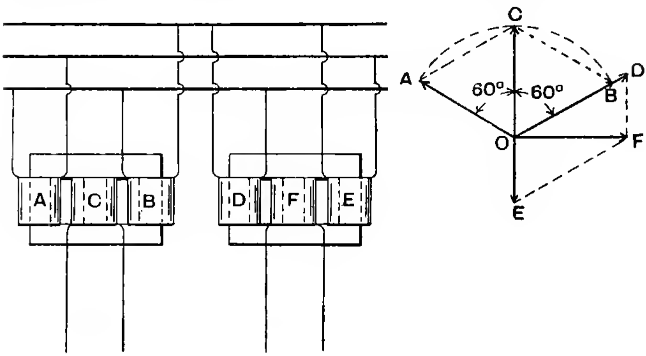


FIG. 57.

Considering first the left-hand transformer, there will be an equal number of turns in the windings A and B, producing equal magnetic fluxes, O A and O B, differing in phase by 120 degrees. The resultant flux through the central core will be O C.

With regard to the right-hand transformer, the number of turns in the coil D will be *smaller*, and in the coil E *greater*, than in coils A or B, the actual number of turns being such as to produce magnetic fluxes in the cores D and E, of the second transformer, proportional to the lengths of the vectors O D and O E. These vectors must

necessarily subtend a phase angle of 120 degrees; but, owing to their *lengths* being suitably proportioned, they will produce a resultant flux, O F, through the central core, F, exactly at right angles to the flux O C, in the core C of the first transformer.

It follows that a two-phase supply can be taken from secondary coils, having the same number of turns, wound on the central cores of the two transformers.

CHAPTER VI

POWER TRANSMISSION BY POLYPHASE CURRENTS

54. ONE of the chief reasons why polyphase currents are used almost to the exclusion of other systems of power transmission and distribution lies in the ease with which such currents can be transmitted over great distances at high pressures to the centres of population, and there converted by means of static transformers to pressures suitable for distribution. When the frequency is not too low—not less than forty cycles per second—both induction motors and incandescent lamps can be connected to the same secondary circuits. If preferred, the three-phase currents, transmitted from a water-power site, or from a coal-mining district, can be used to drive polyphase motors at the receiving sub-stations; and these motors can, in their turn, be coupled to any kind of machine or generator, such as D.C. dynamo machines, if continuous currents are required.

It is not proposed, in this chapter, to consider in detail the many problems arising in connection with the transmission of energy by polyphase currents, not only because the writer has dealt with this subject elsewhere,* but mainly because the scope of this book does not permit of any question of design or of practical details being referred to otherwise than superficially. An attempt

* "Overhead Electric Power Transmission" (McGraw-Hill Book Company).

will, however, be made to put before the reader the essential principles with which the electrical engineer is concerned, involving of necessity some economic considerations; but the mechanical problems, involving strengths of poles and wires, will not be touched upon.

Transmission by single-, two-, and three-phase currents will be considered in succession, the greatest amount of space being devoted to single-phase currents, for the reason that all polyphase transmissions can be treated as a combination of several single-phase transmissions; and a proper understanding of the conditions met with in transmitting single-phase energy over two wires is therefore essential for the solution of problems in the transmission of energy by polyphase currents.

55. **Losses in Transmission.**—With high voltages, such as are necessary for the economical transmission of electric power to a distance, we have to consider, not only the losses due to the heating of the conductors by the current, but also the power lost in the dielectric forming the insulation of the cables, or, in the more usual case of overhead lines, the discharge at the surface of the conductors, and the waste of power in the air. This loss of power accompanies the visible halo of light known as the "corona." It is inappreciable except at the higher voltages; and when the pressure of transmission is below 60,000 volts, it is rarely necessary to take into account the possible increase of power loss due to corona formation.

The actual amount of the losses will depend upon the ratio of the spacing between conductors to the diameter of the wires. It will be directly proportional to the frequency and length of line, and will increase as the *square* of the amount by which the actual voltage exceeds a certain value known as the *disruptive critical voltage*,

this last being practically constant for any given diameter and spacing of wires.

In addition to these two causes of power dissipation, we must not overlook the leakage over insulators on a long high-tension aerial line; but the amount of current which leaks to earth, or between conductors, in this manner is very small on a carefully-designed and well-constructed line, and it is generally permissible to neglect it. The allowable loss of power in a transmission line is a matter of the greatest importance, and it will be again referred to after we have considered the various systems and arrangements of conductors; but for the present a certain percentage loss in transmission will be assumed, apart from economical considerations.

56. **Choice of Voltage.** — Generally speaking, the higher the pressure the more economical will be the transmission, because, for a given total power, the current, and, therefore, the weight of the conductors, is reduced. But there are, obviously, many considerations which stand in the way of very high pressures being used in all cases; and each particular scheme must be examined from every possible point of view before definitely deciding upon the voltage to be adopted. In the first place, a small current at a very high pressure costs more to produce than a larger current at a proportionately lower pressure, either because the generators themselves will be more costly, or because step-up transformers will have to be used. If, however, the line losses would be very considerable with the larger currents, then it might be more economical to lay out capital in step-up transformers, extra insulation of conductors, and step-down transformers at the receiving end, and so save copper in the lines. In other words, the most suitable voltage will depend very largely upon the *distance* to which the energy

has to be transmitted, but also upon the total amount of this energy.

The determination of the proper voltage to be used on any given transmission scheme is mainly an economic question; but, as a rough approximation for preliminary calculations, the following empirical formula may be used:

$$\left.\begin{array}{r}\text{Pressure between wires,}\\ \text{in kilo-volts}\end{array}\right\} = 5{\cdot}5\sqrt{\mathrm{D} + \frac{\mathrm{K.W.}}{150}}$$

—where D = distance of transmission in miles, and K.W. = total kilowatts transmitted.

There are overhead transmission lines actually operating at pressures up to 150,000 volts, and it is probable that the near future will see energy transmitted at pressures in the neighbourhood of 200,000 volts; but the higher pressures should never be adopted without due regard to economical principles.

With reference to underground cables, these are quite out of the question in connection with long-distance transmission by alternating currents. Their cost would be prohibitive, and, apart from this, they are unsuitable for very high pressures. It is unlikely that insulated cables will be used in the near future for pressures much in excess of 50,000 volts, not only because of their excessive cost, but also on account of the very considerable capacity effects and large dielectric losses. There will, however, always be a use for insulated cables at comparatively high pressures, in connection with transmission schemes, because of the objections to carrying the overhead conductors, at extra high pressures, through towns or populous districts.

57. **Transmission by Single-Phase Alternating Currents.**—Let us assume, in the first place, that

not only the load at the distant end, but also the transmission line, are without inductance or capacity.

If E = volts between conductors at generating end;
I = the current in amperes;
R = resistance of "load" at distant end;
r = resistance of each of the two conductors;

then the total power transmitted will be

$$\begin{aligned} W &= E \times I \\ &= I^2 \times (R + 2r), \end{aligned}$$

and the pressure lost in transmission will be $2r \times I$.

These relations are simple, and such as would be obtained if continuous current was being transmitted instead of alternating; but if the conductors are of comparatively large diameter, there will be an inductive effect which, by causing an unequal distribution of current in the conductors, leads to the *apparent resistance*, and the loss of power, being greater with a rapidly alternating current than with one that is steady and unidirectional. This is known as the "skin effect," and it becomes of some practical importance when the conductors are of large diameter, especially if the frequency is high. As a rule the skin effect can be neglected; but in refined calculations the necessary correction should be made.

When a wire carries an alternating current, this current produces a varying flux of induction, not only in the medium surrounding the wire, but also in the space occupied by the material of the conductor itself. This alternating flux has the effect of crowding the current toward the surface of the conductor, with the result that the I R drop of pressure is greater than it would be with a uniform current density. In fact, the result is the same as if a uniformly-distributed current

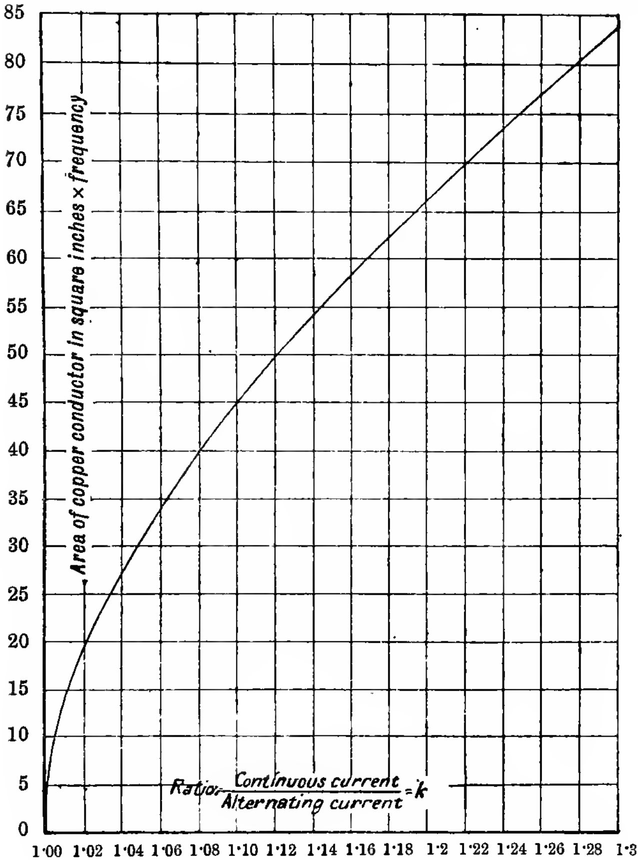


FIG. 58.

were flowing in a conductor of which the resistance is R′ ohms instead of R ohms; and the ratio $\dfrac{\text{apparent resistance}}{\text{actual resistance}}$, or $\dfrac{R'}{R}$ is the "skin effect" coefficient

of which the value, for copper conductors, may be obtained from the curve Fig. 58. This coefficient is a function of the product *section of (circular) conductor* × *frequency*, and this is the quantity which has been used for plotting the vertical ordinates of the curve.

If the conductor is not of copper, but of any other "non-magnetic" material, of circular cross-section, the curve Fig. 58 can be used if the figures on the vertical axis are understood to be the quantity

$$\textit{area} \times \textit{frequency} \times \frac{\textit{conductivity of metal used}}{\textit{conductivity of copper}}.$$

58. **Effect of Inductive Load on Line Losses.**—Let us still assume the line to be without inductance or capacity, and see what is the effect on the line losses if the "load" at the distant end has a power factor less than unity, such as would be the case if the current is supplied to induction motors. Assuming a lag of 37 time-degrees, which corresponds to a power factor (cos 37 degrees) of about ·8, the total current, to deliver the same power at the same pressure, will be 1·25 times greater than if the power factor were unity; and since, for the same loss of power in the line, the resistance multiplied by the square of the current must remain constant, it follows that the cross-section of the conductors must vary inversely as the *square* of the power factor—the proper correction being made for *skin effect* if of sufficient importance. In the case under consideration, the cross-section would have to be increased in the ratio of 1 to ·64, which corresponds to an increase in weight of copper of over 50 per cent. It does not follow that it would be economical to increase the section to so great an extent, but the importance of a high power factor must not be overlooked.

59. **Effect of Taking into Account the Self-Induction of the Lines.**—Since there must be a certain distance separating the outward-going and return wires of the alternating-current transmission line, there must of necessity be an E.M.F. of self-induction produced by the alternating magnetic flux in the space between the

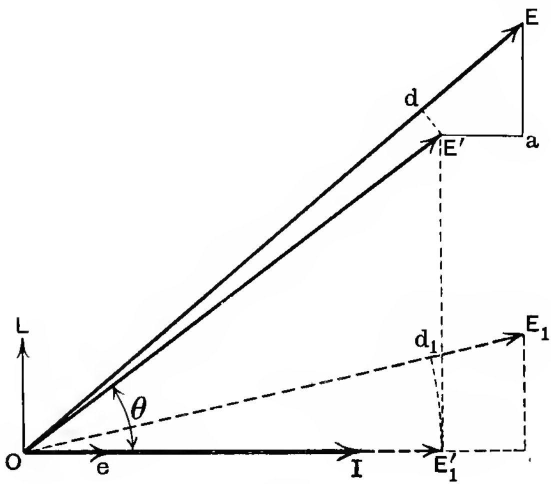


FIG. 59.

wire; and this will be directly proportional to the amount of current in the line.

In Fig. 59, O E′ is the pressure at receiving end, and O I the current, lagging behind the voltage vector by an angle θ. The vector O e, drawn in phase with the current, and equal in magnitude to $2\ r \times I$, is the E.M.F. component required to overcome the resistance of the line. The counter E.M.F. of self-induction will lag behind the

current by a quarter of a period, and the vector, O L, has been drawn 90 degrees *in advance* of the current to represent the component of the total pressure at generating end necessary to balance the E.M.F. of self-induction. The total requisite initial pressure is evidently O E, obtained by compounding the forces O E′, O *e*, and O L. The dotted lines show the effect of resistance drop combined with inductive drop for the same current in the line, and the same total amount of power transmitted, when the power factor at receiving end is unity—*i.e.*, when $\theta = 0$.* A study of this diagram (Fig. 59) leads to the following conclusions:

(1) The additional loss of pressure due to the self-induction of the lines is of considerably greater importance on an inductive than on a non-inductive load.

(2) For a given current and $I^2 r$ loss in the line, the difference in the power factors at the two ends is greatest when the load is non-inductive.

60. **Predetermination of Inductive Drop.**—In the diagram Fig. 59 we have assumed the inductive pressure drop—represented by the vector O L—to be known. In order to calculate it, we must know the coefficient of self-induction of the circuit formed by the go and return wires. This will depend not only upon the distance separating the wires, but also upon their diameter. The greater the distance between wires, and the smaller their diameter, the larger will be the amount of magnetic flux produced by the passage of unit current.

The coefficient of self-induction, L, for *one* conductor

* This method of drawing the diagrams, which assumes the current constant and the pressure variable, to comply with the condition of constant *power* at different power factors is sometimes convenient, but the construction can be modified to suit the condition of a definite initial pressure.

of an overhead transmission line one mile long is given by the formula

$$\text{Millihenrys} = \cdot 741 \times \log_{10}\left(2\cdot 568\,\frac{D}{d}\right),$$

where D is the distance between the centres of the outward and return conductors, and d is the diameter of one conductor; these measurements being expressed in the same units.

Now, the counter E.M.F. of self-induction, in volts, *on the sine wave assumption* is

$$E_L = 2\,\pi\,f\,L \times I,$$

where L is the inductance in *henrys*, and I is the current in amperes.

Inserting the above value of L, we get,

$$\left.\begin{array}{r}\text{Volts induced per mile}\\ \text{of single conductor}\end{array}\right\} = \cdot 004656\,f\,I\,\log\left(2\cdot 568\,\frac{D}{d}\right),$$

which enables us to calculate the length of the vector O L in Fig. 59. It must be understood that, since this formula takes into account the reactance of *one conductor* only, the value of the voltage so obtained must be *doubled* in order to arrive at the total counter E.M.F. of self-induction in the *loop* of a single-phase transmission one mile long. For any other length of line it is, of course, merely necessary to multiply by the distance of transmission expressed in miles.

61. **Capacity of Transmission Lines.**—In the case of a concentric or other underground cable conveying alternating currents, the capacity effects may be very great. A large capacity is generally objectionable in practical working, although the effects of capacity tend

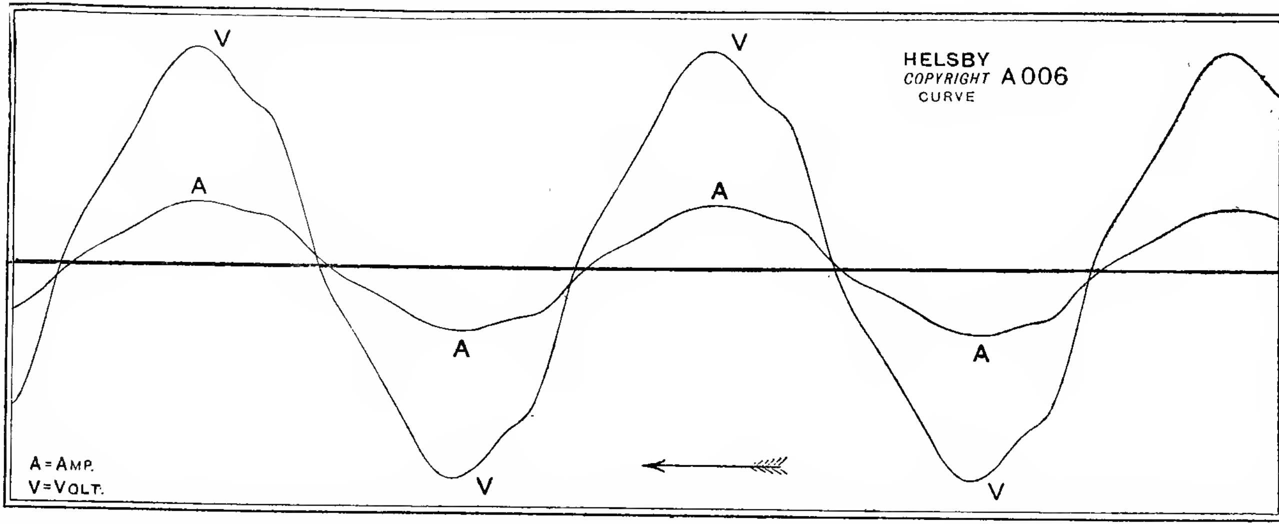
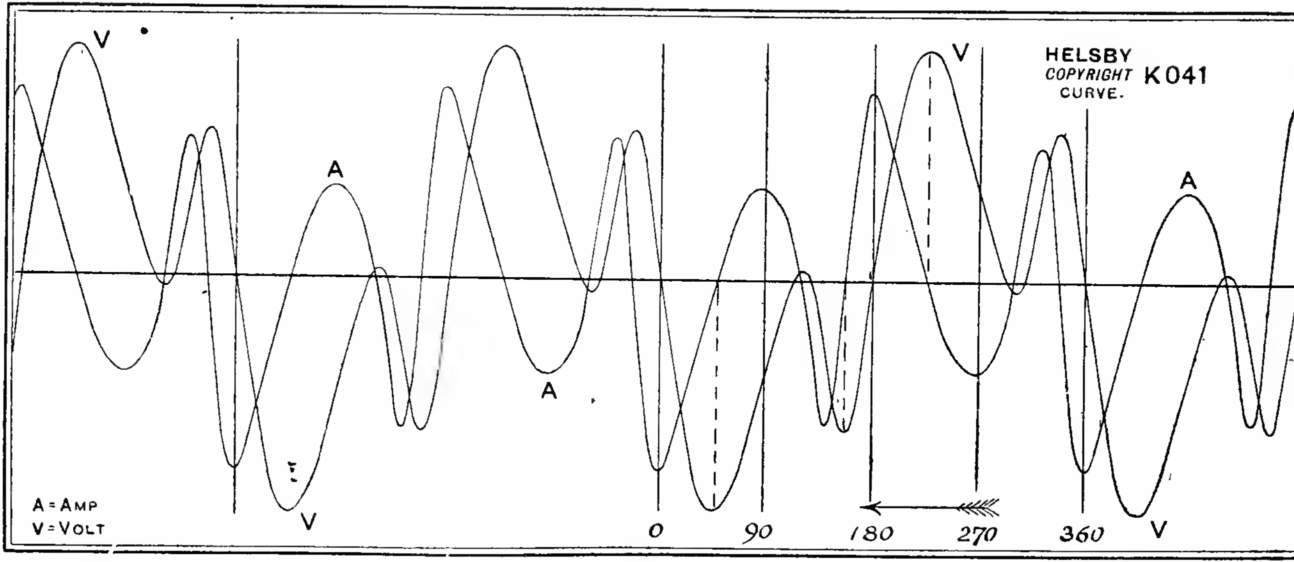


FIG. 60.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A = AMP
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FIG. 61.

to balance those of self-induction, and so bring the power factor nearer to unity. If C is the capacity in microfarads of any system of two conductors, f the frequency, and E the potential difference between the conductors, then the *charging current—i.e.*, the alternating current which will pass between them irrespective of the load at the distant end—will be (see article 26, Chapter II., p. 60)

$$I_c = 2 \pi f C E \div 1{,}000{,}000,$$

on the assumption that the wave form of the E.M.F. is a sine curve. This current is in quadrature with the E.M.F., being 90 degrees *in advance* of the applied potential difference, or 90 degrees *behind* the condenser E.M.F.

The above formula enables us to calculate the capacity current on the assumption of the E.M.F. following the simple harmonic law of variation, provided the capacity, C, can be correctly predetermined, or ascertained by actual measurement; but when we come to consider the various wave forms found in actual practice, it is not possible to calculate the capacity current with a high degree of accuracy. The sine curve wave will be found to give the smallest condenser current of any possible wave form, while irregular wave forms may give rise to capacity effects which it is rarely possible to predetermine with accuracy.

The two diagrams Figs. 60 and 61, reproduced by kind permission of Mr. A. Whalley, of the British Insulated and Helsby Cables, Limited, show the extraordinary distortion of the wave forms due to the introduction of capacity in the circuit of an alternator which does not, under all conditions of load, give a difference of potential at the terminals following the simple harmonic law of variation.

In Fig. 60, V is the pressure and A the current when the alternator is supplying 29·5 amperes at 2,040 volts to a transformer on a nearly non-inductive load; whereas Fig. 61 shows the altered pressure curve and the resulting charging current when the same alternator is supplying a current of 6·04 amperes at 1,980 volts between the two conductors of an insulated cable having a capacity of 4·9 microfarads. It is interesting to observe how the smallest ripple in the original curve may become distorted and magnified. The fact that the maximum values of the E.M.F. (and, therefore, of the charge) invariably occur at the instant when the current is changing its direction should also be noted. (See Fig. 23, Chapter II., article 26).

In the case of overhead lines, the capacity effects are generally small, and sometimes negligible. A convenient formula for use in calculations on overhead transmission lines is

$$C = \frac{\cdot 0194}{\log \frac{D}{r}}$$

where C = capacity in microfarads between the two parallel wires of a single-phase transmission one mile long;

D = distance between centres of conductors, in inches;

r = radius of cross-section of the (cylindrical) wires in inches.

This formula is not scientifically correct, and is not applicable to parallel wires of which the separation, D, is small in proportion to the radius, r; but with the usual separation between conductors on practical over-

head transmissions no greater approach to theoretical accuracy is necessary.

62. **Vector Diagram for Single-Phase Transmission Line, taking into Account Resistance, Inductance, and Capacity.**—In Fig. 62, the generator is shown supplying the total current, I, at the initial pressure, E, through the transmission lines to the distant end, where the load is supposed to be partly inductive. The current delivered at the end of the line is I′, and the pressure E′; the coefficient of self-induction of the transmission lines is 2 L, and their resistance 2 *r*. It will

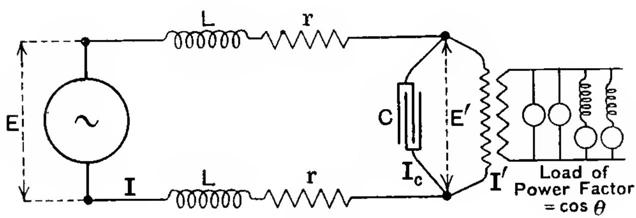

Fig. 62.

be noticed that the whole of the capacity is shown as being concentrated at the end of the line. This assumption leads to a calculated capacity current somewhat in excess of what would be obtained with the same total capacity distributed along the line, but it simplifies the vector diagram. A closer approximation to actual conditions would be obtained by supposing the capacity to be concentrated at the centre of the line (or, perhaps, rather nearer the receiving than the transmitting end); or if still greater accuracy be required, no difficulty need be experienced in drawing the diagram on the supposition that the capacity is due to two or more smaller condensers

connected between the lines at different points. But it must be remembered that it is not possible to calculate the capacity current on a long transmission line with great accuracy, because it will depend largely upon the E.M.F. wave form, which, even if known under certain specified conditions, will generally vary considerably with alterations in the load.

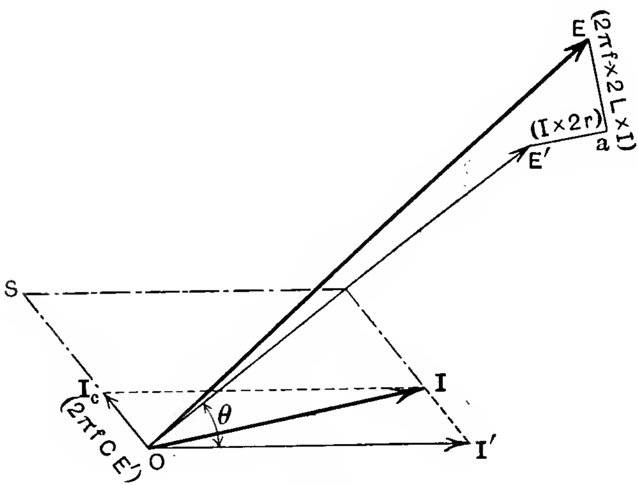

FIG. 63.

Assuming, then, for the sake of greater simplicity of construction, that the total capacity, C, of the line is concentrated at the distant end, we may proceed to draw the vector diagram (Fig. 63). Here O E′ and O I′ represent, respectively, the potential difference and the current at the distant end of the line, the two vectors being drawn with an angle θ between them, such that cos θ is equal to the assumed power factor of the load. Referring to Fig. 62, it will be seen that a condenser, of capacity C,

is supposed to connect the wires at the distant end; the impressed volts at condenser terminals are therefore E′, and the vector for the condenser current (equal to $2\pi f C E'$, on the assumption of the sine curve wave form) must be drawn 90 degrees in advance of O E′. It is represented by O I_c in Fig. 63. The total currrent supplied to the line at the generator end will be O I, obtained by compounding O I′ and O I_c. With regard to the E.M.F. at generator end, this is the resultant of three components—namely: O E′, the pressure available at receiving end; E′ *a* (drawn parallel to O I and equal to $I \times 2r$), to compensate for the drop due to resistance; and *a* E (drawn at right angles to O I and equal to $2\pi f \times 2L \times I$), to represent the pressure required to balance the E.M.F. of self-induction. This gives us O E as the necessary pressure at generating end.

The conclusions to be drawn from this diagram (which takes both self-induction and capacity into account) are as follows:

(1) On an inductive load the current, I, put into the line at the generating end may be *less* than the current, I′, at receiving end.

(2) The total current, I (under the condition of a partly inductive load), comes more and more nearly in phase with the E.M.F. as the capacity current increases up to a certain limit depending upon the power factor of the load. Thus, if the capacity current were equal to O S instead of O I_c, the total current—as indicated by the chain dotted lines in Fig. 63—would be in phase with E′. This illustrates the effect of capacity supplying the magnetising current of the inductive apparatus constituting the load, and so improving the power factor.

The charging current on overhead lines is generally small. Let us take as an example the data which follow:

The pressure $E' = 20{,}000$ volts.
The frequency $f = 25$.
The length of the line = 50 miles.
The diameter of conductors = ·36 in.
Their distance apart = 3 ft.

Using the formula given above for the capacity of an overhead line, we have

$$\text{Capacity in microfarads per mile} = \frac{\cdot 0194}{\log 36/\cdot 18}$$
$$= \cdot 00843,$$

which gives us for the total capacity

$$C = 50 \times \cdot 00843$$
$$= \cdot 4215 \text{ microfarads.}$$

The capacity current will be

$$I_c = 2\pi \times 25 \times \cdot 4215 \times 20{,}000 \times 10^{-6}$$
$$= 1\cdot 3 \text{ amperes,}$$

which is a very small percentage of the total current (assumed to be about 50 amperes). At the same time, we have only to suppose an increase in the length of line from 50 to 200 miles, with a correspondingly higher pressure of, say, 80,000 volts, and imagine the frequency to be 50 instead of 25, in order to obtain a calculated capacity current of nearly 42 amperes, which is by no means negligible. As a matter of fact, the capacity would be somewhat smaller than the value previously calculated, because of the necessary increased spacing between wires.

It will be understood that, where insulated cables are used (only possible for low pressures and short distances), the capacity current may be from twenty to

thirty times as great as in the case of an equivalent overhead transmission, and, moreover, the dielectric losses may become important.

63. **Rise of Pressure at the Distant End of a Long Transmission Line.**—A slight modification of the diagram Fig. 63 will clearly show the well-known effect of a *rise* of pressure occurring at the far end of a line *when the load is very small*, provided also that *both self-induction and capacity are present*. We shall still suppose the arrangement to be as shown in Fig. 62, with the one exception that the transformer and load at the receiving end are entirely disconnected. We shall also suppose the

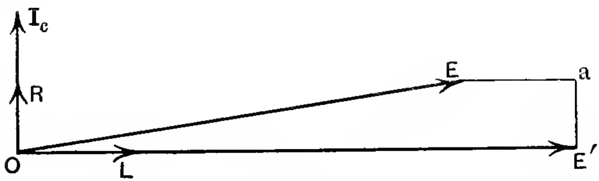

FIG. 64.

capacity and self-induction to be greater than in the previous example, so as to magnify the effect.

Draw O E′ in Fig. 64 to represent the pressure at the distant end; then O I_c (at right angles to O E′—in the forward direction—and equal to $2 \pi f$ C E′) is the capacity current which, under the condition of open circuit at distant end, is also the total current flowing in the wires. We now proceed with the construction exactly as in Fig. 63, making E′ *a* (in phase with the current) equal to $I_c \times 2\, r$, and *a* E (at right angles to the current) equal to $2 \pi f \times I_c \times 2$ L. The vector O E, which, it will be seen, is *smaller* than O E′, is the necessary pressure at generating end. It will be noted that, if self-induction

were absent, the pressure at generating end would be O *a*, which is practically equal in length to O E′; but, since the E.M.F. of self-induction, O L (for this particular state of things), is *in phase with E′*, it stands to reason that the necessary pressure at generating end will be *reduced* accordingly.

Rises of pressure at the end of long underground feeders are by no means uncommon, and they may also occur on long-distance overhead lines. Indeed, a rise of 8 per cent. has been observed on a 40-mile 25,000-volt line when the circuit was open at the distant end. It should be mentioned that the pressure rises due to the combined effects of capacity and self-induction are often greater in practice than would be indicated by the construction of a diagram such as Fig. 64. But this diagram is based on the assumption of the E.M.F. wave being a sine curve, and the abnormal effects sometimes met with in practice are often due to peaked or irregular wave forms. These may lead to enormously increased capacity effects, which, unfortunately, cannot be predetermined accurately.

The calculation of abnormal voltages due to the interruption of heavy currents, or to the condition of *resonance*, would be out of place in this book, and the reader who is interested in these matters is referred to more advanced books, such as "Transient Phenomena," by Dr. C. P. Steinmetz, in which these problems are adequately dealt with. All that the present writer can attempt in these pages is to describe in a few words the conditions that lead to abnormal rises of pressure. In the first place it should be noted that energy can be *dissipated* in a transmission line only in the form of heat, either in the wire itself through ohmic resistance, or in the surrounding air through corona loss, not to mention the

small leakage loss that occurs over insulators. Now, when a circuit has inductance or capacity, or both, energy is stored in the circuit so long as a current is flowing; and this energy must be given up or dissipated when the current ceases to flow. In the case of an alternating current in a circuit of negligible resistance, when the current passes through zero value all the energy is stored in the electrostatic field, and at the instant when the pressure wave passes through zero value this energy is necessarily in the electro-magnetic field. Thus there is an *oscillation* of electric energy, the *frequency* of which can be shown to be

$$f = \frac{1}{2\pi\sqrt{LC}},$$

where L and C are expressed in henrys and farads respectively. This quantity is known as the *natural frequency* of the circuit. It will probably be admitted without proof that, if we impress on the circuit an E.M.F. of which the periodicity is the same as this "natural frequency," abnormal results may be expected. This is the condition of *resonance.* In practice the normal frequency of transmission is much lower than the natural frequency of the line; but the presence of *ripples* in the wave form—which, by analysis, can be considered as "higher harmonics"—may lead to the condition of resonance and abnormal voltages.

The oscillations of energy, previously referred to, in a circuit of negligible resistance, leads to the simple relation

$$\frac{E}{I} = \sqrt{\frac{L}{C}}.$$

This quantity has been named by Dr. Steinmetz the "natural impedance" of the circuit; and it provides a

ready means of calculating the maximum value of any surge pressure due to the sudden interruption of current through the opening of the circuit. It can be shown that, on practical overhead lines, the surge pressure, in volts, cannot exceed about 200 times the amount of the current, in amperes.

64. **Transmission by Two-Phase Currents.**—Let us assume, in the first place, as when considering single-phase transmission, that not only the load at the distant end, but also the transmission line, are without self-induction or capacity.

If we run four conductors as indicated in Fig. 65,

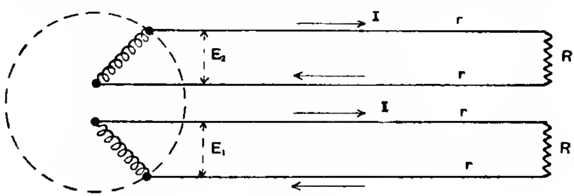

FIG. 65.

keeping the two phases entirely separate, the arrangement resolves itself into the transmission of two distinct single-phase currents; and if the load is equal on both sections, the total power transmitted will be

$$W = 2\,E \times I$$

or, $$2 \times I^2 \times (R + 2\,r),$$

where E = the terminal pressure of each section and I = the current in each section.

The pressure lost in transmission will be $2\,r \times I$, where r is the resistance of a single conductor; and the necessary weight of copper in the conductor, for a given pressure drop (or loss of power), will be the same as for

a single-phase transmission scheme. It is, however, evident that, by combining two of the conductors to form a common return, the transmission of two-phase currents can be effected with only three wires, as shown in Fig. 66.

The vector diagram for such a system of transmission is very simple, *if we can assume the resistance, r', of the common conductor to be negligible*, and it has been drawn in Fig. 67. Here the phase difference of a quarter period between the two E.M.F.s is represented by E_1 being drawn 90 degrees in advance of E_2, and since the

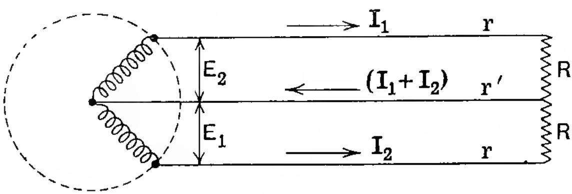

FIG. 66.

circuits are balanced—*i.e.*, since the load is the same on both phases—we may write

$$I_1 = I_2 = \frac{E_1}{r + R} = \frac{E_2}{r \times R};$$

and as we are not taking into account either capacity or self-induction, the two currents will be in phase with the respective E.M.F.s, and the return current in the common conductor will be $\sqrt{2}$ times I_1 or I_2.

When the common return wire has appreciable resistance, as would be the case on a power transmission scheme by two-phase currents, complications arise which are accentuated by the unsymmetrical phases of the counter E.M.F.s of self-induction. The result is that,

even when the impressed E.M.F.s are exactly equal, with a phase difference of a quarter period, the currents in the two phases may not be equal, and the phase angle between them is usually greater than 90 degrees. These reasons account for the fact that two-phase currents are almost invariably transmitted over four wires. Taking into account the additional fact that

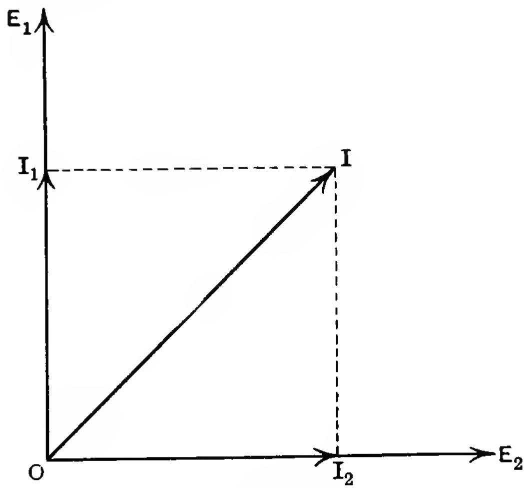

FIG. 67.

transmission by three-phase currents has now been adopted almost to the exclusion of two-phase currents, it appears unnecessary to investigate further the reasons leading to this loss of symmetry under load conditions, and we will pass on to the consideration of power transmission by three-phase currents.

65. **Transmission by Three-Phase Alternating Currents.**—We shall again assume, in the first instance,

that the line is without self-induction or capacity. If we were to run six separate conductors—*i.e.*, two for each phase—we should merely be transmitting three independent single-phase currents, the arrangement being as shown in Fig. 68; and if

e = the potential difference at the terminals of each circuit;
I = the current in each wire;
r = the resistance of each wire;
R = the resistance of the "load" on each phase,

then the total power transmitted would be

$$W = 3\,(e \times I),$$

or,

$$W = 3 \times I^2 \times (R + 2\,r).$$

The pressure lost in transmission would be $2\,r \times I$, and the total power lost in the lines would be $3 \times I^2 \times 2\,r$.

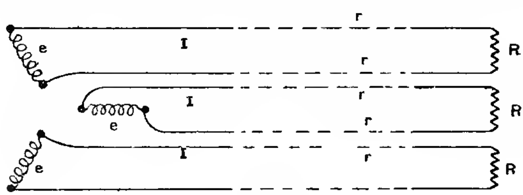

FIG. 68.

Let us now see what is the effect of providing a common return for these three circuits. The arrangement is shown in Fig. 69.

The pressure at generating end between the three terminals of the alternator and the common return, or neutral point, is still e volts, and the total power transmitted is still $W = 3\,(e \times I)$; but, owing to the fact that the sum of the three outgoing currents is zero (seeing

that they differ in phase by 120 degrees, as shown in Fig. 70, and that any one current, such as O B, is exactly equal and opposite to the resultant of the other two currents), there will be no current flowing in the common return conductor, and it follows that both pressure drop and $I^2 r$ losses in the lines are reduced to *one-half* of what they were with the arrangement of three separate circuits, the power loss in the lines being now $3 I^2 r$. This clearly shows how the transmission by three-phase currents is more economical as regards line losses than single-phase transmission. But it must not be over-

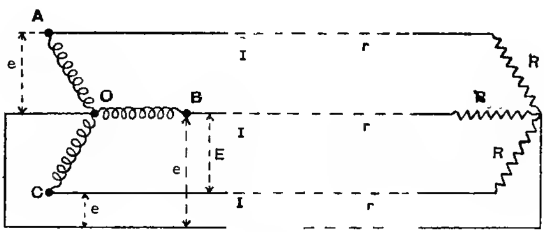

FIG. 69.

looked that, in order to obtain a reduction by half of the weight of copper in the lines, the pressure between wires is greater on the three-phase system than on a single-phase system transmitting the same power. Thus, the pressure, E (Fig. 69), between any two of the three transmission wires is equal to $e \times \sqrt{3}$, as shown by the diagram Fig. 71. Here the E.M.F.s in the three sections of the alternator windings are represented by O A, O B, and O C; and since the E.M.F., E, between any two terminals, such as B and C (Fig. 69), is the resultant of the E.M.F.s acting (away from the common junction O) in the two windings O B and O C connected in series,

we must *subtract* one of these (the vector O C) from the other (the vector O B). Thus the resultant, O E (Fig. 71), is obtained by *adding* to the vector O B an imaginary vector, O C′, exactly equal but opposite to O C. This resultant is evidently equal and parallel to the line, C B, joining the ends of the two vectors O B and O C, and it can easily be shown to be exactly $\sqrt{3}$ times greater than either of these vectors. We may, therefore, write

$$E = 1{\cdot}732\, e.$$

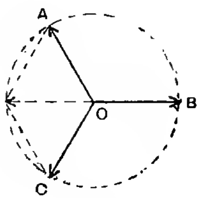

FIG. 70.

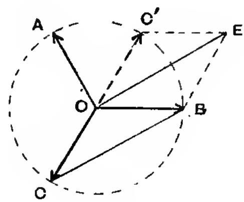

FIG. 71.

The *power* of a three-phase circuit, which is equal to three times $e \times I$, can evidently also be written

$$W = 3\left(\frac{E}{\sqrt{3}} \times I\right)$$

or,

$$W = \sqrt{3}\, E \times I,$$

where E is the pressure between any two of the three wires.

In comparing three-phase with single-phase transmission *on the basis of the same maximum potential difference between wires,* we have, for the total three-phase power

$$W = \sqrt{3}\, E \times I,$$

which, for equal power, single-phase, would be written

$$W = E \times (\sqrt{3}\, I).$$

Let r = the resistance of each conductor on the three-phase system, and

r_1 = the resistance of each conductor on the single-phase system,

then *weight of copper, single phase* $\propto 2 \times \frac{1}{r_1}$

and *weight of copper, three phase* $\propto 3 \times \frac{1}{r}$

and the ratio

$$\frac{\text{weight of copper, single phase}}{\text{weight of copper, three phase}}$$

may be written $\dfrac{2\, r}{3\, r_1}$.

But for equal line losses we have

$$3\, I^2 r = 2\, (\sqrt{3}\, I)^2 r_1,$$

which gives us $r = 2\, r_1$.

Hence the relative weights of copper are as 4 : 3, showing a saving of 25 per cent. in favour of the three-phase system.

66. **Effect of Inductive Load on Line Losses.**—We shall still neglect the capacity and inductance of the lines, but briefly consider the effect of an inductive load on the vector diagrams.

In Fig. 72, the diagram has been drawn for the condition of a power factor of unity.

Here O A, O B, and O C are the three current vectors,

The potential difference between wires is

$$a\,b = b\,c = c\,a = \text{E}.$$

The length $\text{O}\,a = \text{O}\,b = \text{O}\,c = e = \dfrac{\text{E}}{\sqrt{3}}$.

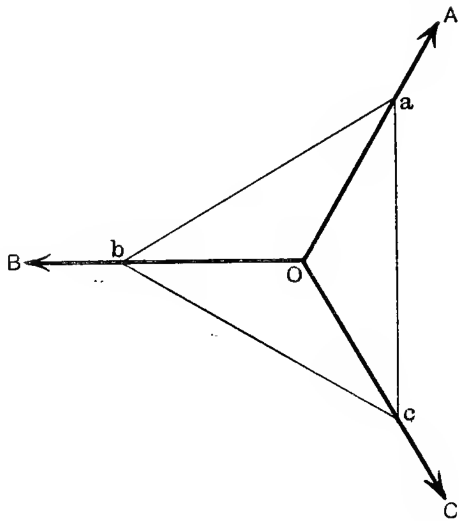

FIG. 72.

The total power is:

$$\begin{aligned} \text{W} &= \sqrt{3}\,\text{E} \times \text{I} \\ &= 3\,e \times \text{I} \\ &= 3\,(\text{O A} \times \text{O}\,a). \end{aligned}$$

In Fig. 73, the diagram has been drawn for an inductive load.

Here there is a certain displacement of the current phases relatively to the E.M.F. phases. It will be

noticed that the vertices of the E.M.F. triangle no longer lie on the current lines as in the previous diagram. The three current vectors still make the same angle of 120 degrees with each other, but they have been moved bodily round (in the direction of retardation) through an

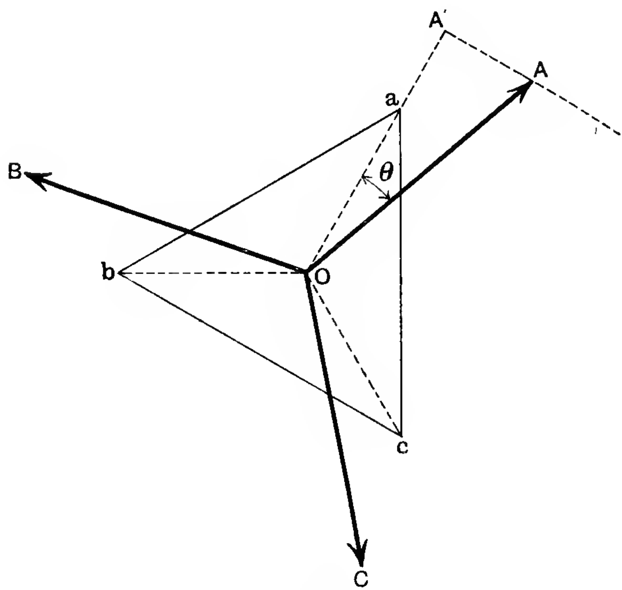

FIG. 73.

angle θ. The total power is, evidently, no longer equal to three times O A $\times$ O a, but to 3 O A′ $\times$ O a, where O A′ is the projection of O A on O a; and cos θ is the power factor of the three-phase circuit.

It was explained in connection with single-phase currents that, for a given total power to be transmitted

at a definite pressure, the weight of the conductors—if the $I^2 r$ losses are to remain constant—must vary inversely as the square of the power factor. This rule necessarily applies equally to the case of three-phase transmission. Thus, in Fig. 73, the power factor ($\cos \theta$) is expressed by the ratio $\frac{O A'}{O A}$; but, if the *power* and *pressure* are constant, the length O A′ will not alter, whatever may be the power factor. Hence O A (the current in any one of the wires) will vary inversely as the power factor, and—for a given loss in the line—the cross-section of the conductor must vary as the square of the current, *i.e.*, *inversely*, as the square of the power factor.

67. **Self-Induction of Three-Phase Lines.**—The arrangement of three-phase overhead lines is generally such that the wires occupy the vertices of an equilateral triangle. Under such a condition, it is evident that the magnetic flux due to one of the wires will neither increase nor decrease the amount of the induction through the loop formed by the other two wires. As a matter of fact, if the three wires are arranged in any other practical manner, the effect of the induction due to any one wire on the loop formed by the other two wires is generally negligible.

The currents in the three wires are equal; but they differ in phase by 120 degrees. If, therefore, we calculate —by means of the formula in article 60, p. 151—the induced E.M.F. on the assumption that each wire produces its own flux of magnetism independently of the other wires, we have merely to combine two such E.M.F.s in the manner previously indicated (see Fig. 71) to obtain the total E.M.F. in any one of the three loops. This is evidently $\sqrt{3}$ times as great as the induced E.M.F. calculated for the current in one of the wires only.

By far the simplest way of constructing the vector

diagrams for three-phase circuits is to consider the three phases independently, and Fig. 74 shows the effect of taking into account the self-induction and ohmic resistance of the lines.

Here A, B, and C are the current vectors, and *a′ b′ c′*

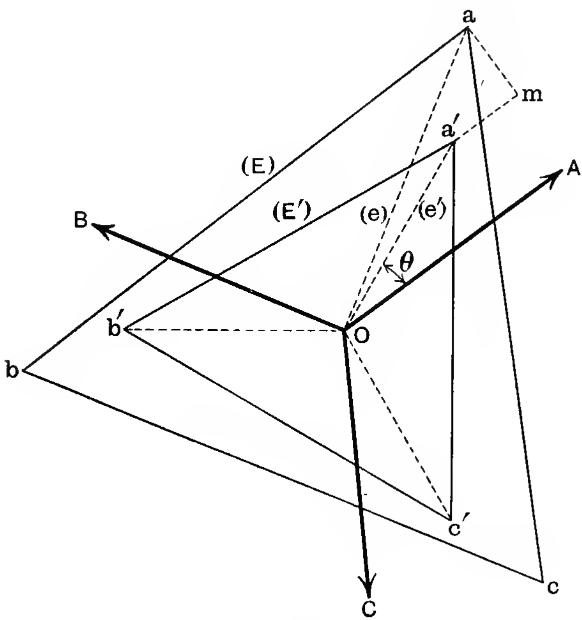

FIG. 74.

is the E.M.F. triangle for the receiving end; the angle of lag being θ.

Consider O A and O *a′* as being the current and E.M.F. vectors of a single-phase circuit. Draw *a′ m* parallel to O A to represent the ohmic drop due to the resistance of one conductor, and *m a*, at right angles to

O A, to represent the necessary impressed voltage to balance the E.M.F. of self-induction, on the assumption that the magnetic field is due solely to the current, O A, *in one conductor*. Join O a, and note that, so far, the construction is exactly similar to that adopted in Fig. 59 for a single-phase circuit. We have merely to suppose the same construction to be followed in the case of the remaining two phases, and the result is the triangle $a\ b\ c$, which represents the necessary three-phase applied E.M.F. at the generating end.

The conclusions drawn from the diagram Fig. 59 when treating of single-phase currents are equally applicable to the case of a three-phase transmission.

Assuming the distance between the wires to be the same for a three-phase as for a single-phase transmission, let us see whether either system has an advantage over the other from the point of view of inductive drop.

In the first place, the total power delivered on the three-phase system (see diagram Fig. 74) is

$$\begin{aligned} & 3\,\mathrm{O\,A} \times \mathrm{O}\,a' \times \cos\theta \\ &= 3\,\mathrm{I} \times e' \times \cos\theta \\ &= \sqrt{3}\,\mathrm{I} \times \mathrm{E}' \times \cos\theta, \end{aligned}$$

where I = the current in any one conductor and E′ = the pressure between wires at receiving end.

In a single-phase system supplying the same power at the same pressure between wires, the current is, therefore, $\sqrt{3}$ times as great.

The inductive drop in one of the three-phase *loops* is proportional to $\mathrm{I} \times \sqrt{3}$, and exactly equal to the inductive drop due to each of the single-phase *wires*; hence, if the diameters of the wires were the same in both cases, the inductive drop in a three-phase line would be only

half that in an equivalent single-phase line. As a matter of fact, the diameter of the conductors in the single-phase line would be greater, on account of the larger current, and this has the effect of *reducing* the self-induction on the single-phase line by something under 10 per cent.

68. **Capacity in Three-Phase Transmission Lines.**—The capacity of a three-phase line with the wires arranged in the form of an equilateral triangle may be considered as being made up of three equal capacities, all measured between any one wire and the line of zero potential, which may be taken as a line passing through the centre of the triangle. The three capacities are shown diagrammatically in Fig. 75.

The capacity of each of the three imaginary condensers shown in the diagram may be calculated by the formula:

$$C = \frac{\cdot 0388}{\log_{10} \frac{D}{r}}$$

where C = the capacity *per mile* in microfarads;
D = the distance between wires;
r = the radius of any one wire.

It follows that the capacity current per conductor can readily be calculated exactly as explained in connection with single-phase transmission, the condenser E.M.F. being taken as the pressure between the wire and neutral line, which is equal to $\frac{1}{\sqrt{3}}$ times the pressure between wires.

This method of calculating the capacity effects on a three-phase line is very simple and convenient; it is evidently based on the assumption that the influence

of the earth may be neglected; and this assumption is justified if the distance between the wires is small as compared with their height above ground.

The formulæ for the capacity of three-core cables with lead coverings are not so simple as for overhead wires, and information regarding the amount of capacity current to be expected in a given type of cable should be obtained from the maker.

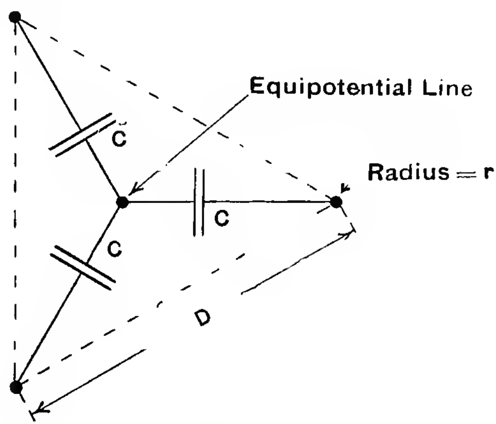

FIG. 75.

69. **Most Economical Section of Transmission Lines.**—In the examples previously worked out, we have assumed certain $I^2 r$ losses in transmission which have enabled us to draw certain conclusions as regards pressure drop, phase displacement, etc. We shall now briefly consider the chief points to be borne in mind when determining the most economical size of conductors for any given conditions of electric transmission.

The question we have to solve is simply this: given a

certain system of transmission (whether single- or polyphase), will it pay us better to put in large conductors at a high initial cost, but having small $I^2 r$ losses, or small conductors at a lower initial cost, but having large $I^2 r$ losses? There is evidently a particular size of conductor which will prove to be the most economical to use on any given transmission scheme; but the difficulties of correctly estimating the amount and duration of load are generally considerable, and we have to be satisfied with a more or less close approximation to the best results.

Again, it will sometimes be found that the section of conductor, as worked out in accordance with economical considerations, cannot be adopted in practice for either or all of the following reasons:

(1) The current density may be so high as to raise the temperature beyond the safe limit.

(2) The pressure drop on a long line may be greater than can conveniently be dealt with.

(3) In the case of overhead wires, the section may be so small that the line would be mechanically weak.

(4) In the case of extra high pressures, the surface of the wire may be so small as to lead to excessive corona loss.

The corrections to be made in such cases are fairly obvious: the section of the conductor must be increased to the required amount without regard to the question of economy; or it may be found that aluminium conductors can be adopted with advantage. If the cables are insulated and laid underground, and the economical current density is so high as to raise the temperature unduly, two or more cables will have to be laid side by side and connected in parallel, or the cross-section of the single cable will have to be increased, according to which method proves to be the cheapest.

Kelvin's law, in its simplicity (and, indeed, when the

law of maximum economy departs from this simplicity, it ceases to be Kelvin's law), may be stated as follows: "The most economical section of a feeder is that which makes the annual cost of the $I^2 r$ losses equal to the annual interest on the capital cost of the copper in the line, plus the necessary annual allowance for depreciation." The cross-section should, therefore, be determined solely by the current which the conductor has to carry, and not by the length of the line, or an arbitrary limit of the percentage full load drop in pressure; the drop, even if considerable, must take care of itself. If there are reasons which make a large drop undesirable, then, if necessary, economy must be sacrificed, and the line calculated on the basis of pressure drop only.

Kelvin's law is based on the assumption that bare conductors are used, and that the cost of these, erected in position, is directly proportional to the weight of the copper. This assumption is not justified in practice if insulated cables are used, the cost of which, per pound of copper, is greater for the small than for the large sizes. But, in the case of an overhead transmission line with bare conductors, Kelvin's law may give sufficiently accurate results; because, although the cost of erecting the smaller wires may be greater *per pound* than for the larger sizes, the insulators, supports, etc., may become more costly as heavier wires are used, thus making that portion of the total cost of the line *which depends upon the section of the conductor* approximately proportional to the weight of the same.

To cover all cases in practice, the law of maximum economy may be stated as follows: *The annual cost of the energy wasted per mile of the transmission line, added to the annual allowance (per mile) for depreciation and interest on first cost, shall be a minimum.*

It is not an easy matter to estimate, even approximately, the probable amount of the $I^2 r$ losses during the year's working. The energy lost in a given conductor depends upon the square of the current and the time during which the current is flowing. If a cable conveys a current I only twelve hours out of the twenty-four hours, then the energy wasted per day is only half what it would have been had the current been flowing continuously, and the watt-hours may be expressed as $12 \times I^2 \times r$, or as $24 \times \frac{I^2}{2} \times r$: it follows that, in working out the energy lost per annum in the conductor, we have to multiply the resistance, not by the square of the maximum current which the conductor will at times be carrying, but by the mean value of the square of the current throughout the year. The expected average daily load curve should therefore be drawn, and the mean value of I^2 calculated therefrom. If the capacity current is considerable, this must be taken into account when calculating the average value of the square of the current. The diagram Fig. 76 will serve to explain how the proper size of conductor may be calculated.

Let us suppose that 10 per cent. is to be allowed for depreciation and interest on cost of conductor; then, in the diagram Fig. 76—where the horizontal distances represent resistance per mile, and the vertical distances represent money—plot the curve A, which gives the relation between 10 per cent. of the capital spent on the conductor, and its resistance in ohms per mile. Now calculate the cost of the $I^2 r$ losses per mile of cable, for any particular value of the resistance, and draw the straight line O B through the origin, which will give us the cost for any other resistance of conductor.

By adding the ordinates of the curves A and B, the

curve C is obtained, of which the minimum value corresponds with the resistance per mile of the conductor which will be the most economical to use, whatever may

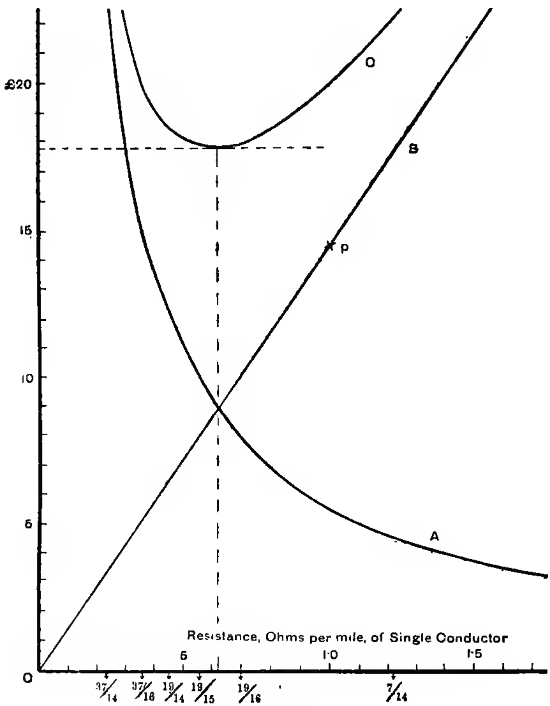

FIG. 76.

be the length of the line or the pressure required at the receiving end.

Let us briefly consider how the two costs, of which the sum is to be a minimum, should be estimated.

In the first place, the capital outlay on which the interest is calculated must include the insulation on cables (if any) and the difference in cost (if any) of the conduits or overhead construction *which is dependent upon the size of the conductors.* No capital charge which is *constant* for all sizes of conductor within practical limits need be taken into account, because the effect of adding the *same length* to all the ordinates of the curve A (Fig. 76) will be to raise the curve C by the same amount, *without altering either its shape, or the position (in the horizontal direction)* of its point of minimum value.

With regard to what is a reasonable percentage to allow for interest on cost of conductors, and depreciation, this will have to depend on commercial considerations and the estimated "life" of the conductors; also on the probable scrap value of the metal when it is replaced.

The second of the two-cost items—namely, the cost per annum of the energy wasted in the line—is more difficult to estimate. It will generally be the works cost of the power; but it must not be overlooked that there are conditions under which the cost of power wasted in the line is almost negligible. Such a condition occurs in connection with hydro-electric schemes, when the demand for power is small in relation to the power available. On the other hand, if the demand exceeds the available supply, the cost of the wasted power is actually the selling price. This was pointed out by Professor George Forbes in his paper on "Distant Electric Power Transmission."*

Example illustrating Use of Diagram.—Consider an overhead line with bare copper conductors, to carry a maximum current of 60 amperes. Let us assume that probable load curves have been drawn, which will enable us to

* *Journ. Inst. E. E.*, vol. xxix., May, 1900.

estimate the $\sqrt{\text{mean square}}$ value of the current throughout the year. This might be about one-third of the maximum value; let us say 20 amperes. If, therefore, we multiply the resistance of each conductor by the square of 20, we shall obtain the total power lost in the line on the assumption that this loss is going on day and night throughout the year. This gives us as the total number of Board of Trade units wasted in each conductor per annum *for an assumed resistance of* 1 *ohm :*

$$\text{Units} = \frac{1 \times (20)^2 \times 24 \times 365}{1{,}000}$$
$$= 3{,}500;$$

and if the cost of this power be taken at 1d. per unit, the cost would be

$$\frac{3{,}500}{12 \times 20} = £14{\cdot}6.$$

This enables us to plot the point p on the curve O B, and, by joining O p, we can read off the cost of power per annum for any other resistance of conductor.

With regard to the curve A, we shall suppose that 10 per cent. is to be allowed for depreciation and interest on capital spent on conductors; and, if we take the cost of these at 1s. 3d. per pound, we can readily plot a number of points through which this curve must be drawn. The curve C, as already explained, is the result of adding the ordinates of curves A and B. Its minimum value, in our example, corresponds to a resistance of ·62 ohm per mile, which gives us as the most economical conductor a stranded cable somewhere between $^{19}/_{16}$ and $^{19}/_{15}$, or the equivalent solid wire. (It should be noted that—since the curve A is a rectangular hyperbola—the minimum value of C is on the same ordinate as the point

where the two curves cross—*i.e.*, where A is equal to B; but this is not necessarily the case when the conductors are provided with an insulating covering.)

It may be well to point out that the graphic method of applying Kelvin's law is used here mainly because it is helpful in explaining the principles upon which the economic conductor section is determined. Other methods of calculation would usually be adopted by the practical engineer; and these calculations, together with the broader aspects of transmission line economy, are discussed in the writer's book on Overhead Power Transmission: it would be out of place to go further into these matters here.

70. **Relative Advantages of Different Systems of Transmission.**—By way of introduction, it should be mentioned that a comparison of various systems on general lines is not only useless, but impossible: there is no basis of comparison applicable to all conditions of practice, and each particular case must be considered on its own merits.

The following points should not be overlooked:

(1) In comparing the relative cost of conductors necessary for two different systems of transmission, the *efficiency* must be the same in both cases—*i.e.*, the same amount of energy must be considered as being transmitted the *same distance* with the *same loss*.

(2) What is to be understood by the *pressure* must be clearly defined. For instance, when comparing continuous with alternating current transmission, it must not be forgotten that the stress on the insulation is less with continuous currents than with alternating currents at the same $\sqrt{\text{mean square}}$ pressure. In fact, on the assumption of a sine curve wave, it is easy to see that—for the same power transmitted, at the same efficiency, and *with*

the same maximum stress on insulation—the weight of copper for a single phase alternating-current transmission would be *double* that required if continuous currents were used.

Again, when considering alternating-current systems, it is necessary to decide whether it is the maximum potential above earth or the maximum pressure between wires which should be taken as a basis of comparison.

Assuming the middle, or neutral point, of the various systems to be "earthed," and allowing for a definite maximum pressure between any wire and earth, the comparison between the different alternating-current systems becomes a simple matter.

Let e and I stand respectively for the voltage and current *per leg* in either of the three systems shown diagrammatically in Fig. 77. Then the total power transmitted in each case will be

$$e \, \mathrm{I} \cos \theta \times n,$$

where $\cos \theta$, as usual, is the power factor, and $n =$ the number of legs.

If r be the resistance of each line conductor, the total line loss will be

$$\mathrm{I}^2 \, r \times n,$$

and, for the same line efficiency, the weight of copper per horse-power transmitted will evidently be the same in all cases.

This leads us to the conclusion that, *for any polyphase system*, the power lost in the line depends only upon the joint resistance of the conductors, the power transmitted, and the power factor, *provided the pressure between any wire and the neutral point is constant.*

Let us neglect the capacity and self-induction of the

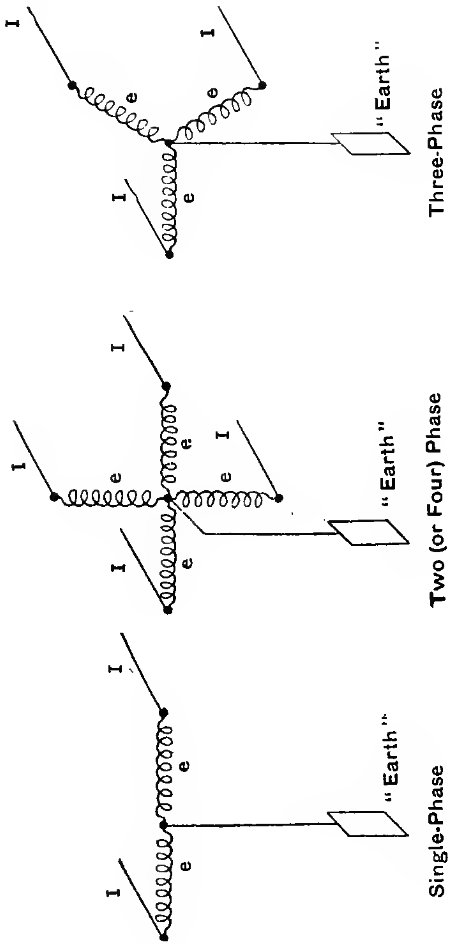
Three-Phase
Two (or Four) Phase
Single-Phase
"Earth"
"Earth"
"Earth"
I
e

FIG. 77.

line, and assume that $\cos\theta$ is the power factor of the load at receiving end. Then

$$\textit{power delivered} = n\,e\,I\cos\theta = W,$$

and the *power lost in line* $= n\,I^2\,r$.

This last quantity, expressed as a percentage of the power delivered, becomes

$$\textit{percentage power lost in transmission} = \frac{n\,I^2\,r \times 100}{W}.$$

Let us denote by R_p the joint resistance of all the line conductors in parallel—*i.e.*, $R_p = \frac{r}{n}$. This gives us, for the numerator of the above percentage loss, the expression

$$n^2\,I^2\,R_p \times 100.$$

But $n^2\,I^2$ is equal to $\frac{W^2}{e^2\,(\cos\theta)^2}$.

Hence we may write:

$$\left.\begin{array}{l}\text{Percentage power lost in trans-}\\ \quad\text{mission for } \textit{any} \text{ balanced}\\ \quad\text{polyphase system}\end{array}\right\} = \frac{W \times R_p}{e^2\,(\cos\theta)^2} \times 100.$$

It might at first sight appear as if there were nothing to choose between the various systems shown in Fig. 77; but it must not be forgotten that the potential difference *between the wires* has not been taken into consideration. When this is done, the advantage will be clearly seen to be in favour of the three-phase system; thus, the maximum pressure between any two of the line wires will be 1·73 e for the three-phase system, and 2 e for the single- or four-phase system.

Although we have assumed the case of an earthed neutral, there is much to be said in favour of a system without an earthed point, mainly as regards reduced risk of stoppage through a breakdown of insulation.

Where current for power and lighting is to be taken off the same mains, the two-phase system, with the phases kept entirely distinct, appears to offer some advantages; or the power may be transmitted by three-phase currents and converted into two-phase by a suitable arrangement of transformers.

For the transmission of energy to a distance by means of alternating currents, the three-phase system would, on the whole, appear to have advantages; and it has been adopted on the continent of America almost to the exclusion of other systems. On the continent of Europe, and, quite recently, in this country, power transmissions have been carried out with continuous currents on the Thury system, which, in special cases, compares very favourably with the more usual polyphase transmissions.

71. **Arrangement of Overhead Lines in Practice.**—It is not intended to deal with any questions concerning the practical construction of transmission lines, or the calculation of stresses and strains in wires or supports; but this chapter would not be complete without some mention of various minor matters affecting the design of transmission lines from an electrical standpoint.

Distance between Wires.—The spacing of conductors on overhead polyphase transmission lines depends on the voltage and the length of span. A convenient rule for spacing, which recent practice confirms as being reasonable and satisfactory, is to add 12 to the pressure between wires expressed in kilovolts; this gives the distance between wires in inches for spans up to 200 feet. For

greater spans, up to a limit of 800 feet, add another 5 inches for every 100 feet length in excess of 200.

Clearance to Pole or Tower.—This should not be less than 9 inches for pressures up to 10,000 volts. The clearance would be increased to about 18 inches at 35,000 volts, and 24 inches at 70,000 volts.

Clearance above Ground.—The minimum clearance between overhead conductors and ground is usually about 20 feet on the lower voltage lines, such as would be carried on wood poles; and 30 feet or more on the extra high pressure lines carried on steel towers. The actual height of the point of support will depend on the length of span and the diameter of the conductor, also upon whether copper or aluminium is used, as these are factors which influence the "sag" of the suspended wire. The span will usually be between the limits of about 150 feet on wood pole lines, and 400 to 600 feet on steel tower lines; but considerably greater spans are used at river crossings or to meet special conditions.

Aluminium Conductors.—For the same conductivity as copper, the diameter of an aluminium wire will be about one and a quarter times as great. The *weight* for the same conductivity is almost exactly *half* that of copper, but the larger area exposed to high winds must be taken into account in making strength calculations. In countries where the price of aluminium is not controlled for the benefit of the copper market, the cost of a transmission line usually works out lower with aluminium than with copper conductors; but it does not necessarily follow that the cheaper metal should be adopted. Hard-drawn copper is an excellent material for overhead lines, and it is mechanically stronger than aluminium. The relative merits of the two metals should be carefully considered in connection with each particular power scheme. A

very large number of important undertakings have adopted aluminium in preference to copper.

Insulators.—These are usually of the "pin" type for pressures below 40,000 volts, and of the "suspension" type for the higher voltages. In the "suspension" type a number of separate unit insulators are strung in series between the point of attachment and the conductor, which is suspended *below* the cross-arm, instead of being *above* the cross-arm as with the pin type insulator.

Porcelain is the material most commonly used for insulators, but glass is used occasionally for the lower pressures. The design of insulators for the higher voltages cannot be undertaken without thorough study; and modern development in the manufacture of insulators is largely the result of experience. Space does not permit of any discussion of the principles of design, which are the same for insulators used on polyphase systems as for those used on any other high-voltage transmission. It may be mentioned, however, that the problems facing the designer are concerned mainly with the proper distribution of *electrostatic capacities* between the various parts of a high-pressure insulator. The leakage resistance, whether of surfaces or of the material of the insulator, is a secondary matter.

Protection against Lightning and Abnormal Pressure Rises.—It is usual to instal lightning arresters at the generating end of the line, and also at the receiving stations. On long lines arresters are also sometimes distributed at intervals, as, for instance, at junctions or switching points. It is difficult to protect the line itself from a direct stroke of lightning, but damage from this cause is rare. An earthed "guard wire"—which may be a stranded galvanised steel cable about $\frac{5}{16}$ inch in diameter —joining the tops of the poles or towers, and carried the

whole length of the line above the conductors, affords good protection, and is extensively used, although some engineers object to its use on the grounds of expense and the possibility of the wire falling across the conductors and causing interruption to service. If the guard wire is not used, lightning rods should be provided on poles or towers at least every 500 feet. The insulators themselves may be protected by "arcing rings," which are so placed that an abnormal rise of pressure will leap across clear of the porcelain, and so prevent damage due to the heat of the arc. The main object of lightning protective devices is, however, to prevent damage to apparatus in generating- and sub-stations.

The ordinary type of horn gap arrester may give very good results if properly installed, with a non-inductive resistance in series with the ground connection; but it does not cover the whole field of lightning protection, and shares with all spark-gap devices the disadvantage that it is liable to set up surges or high-potential disturbances in the circuit.

The *multi-gap*, or so-called "low equivalent," arrester consists of many small air gaps between cylinders of "non-arcing" metal, all in series. It gives good results on circuits up to about 30,000 volts.

For the higher voltages the aluminium cell arrester is extensively used, especially in America; and it certainly seems to afford reasonably good protection when properly installed. A number of aluminium cups or trays, separated by an electrolyte, are built up in the form of a column which forms a connection between line and ground. A film of hydroxide of aluminium is formed on the plates, and this resists the flow of current until a certain critical voltage is reached, which punctures the film at a multitude of points, allowing a fairly large

current to pass to ground; but so soon as the discharge has taken place, and the pressure returns to normal, the film re-forms, and is ready to take another discharge.

European methods of lightning protection—which cover those installations carried out abroad to the specifications of European engineers—differ considerably from American practice; but the time is hardly ripe for passing an unbiassed judgment in favour of either European or American practice. The reason of these differences of method lies probably in the fact that the American engineer has to take—or prefers to take—what the one or two controlling manufacturing concerns choose to offer him; while the European engineer is at liberty to make full use of his knowledge, experience, and judgment. It is not suggested that the manufacturing monopolies of America are not "up-to-date" in the electrical apparatus they put on the market, but merely that they set the fashion in the appliances used in their country. It should, indeed, be mentioned that Professor E. E. F. Creighton, Consulting Engineer to the General Electric Company, is one of the greatest authorities on lightning protection; and those who wish to pursue this subject further are referred to his writings.

Mutual Induction—Transpositions.—If a three-phase line is arranged so that each wire in turn occupies the central position over a distance equal to one-third of the entire length of line, it should have no inductive effect on neighbouring parallel conductors. The influence of one power circuit on a neighbouring power circuit, even if carried on the same set of poles, is usually negligible; and the transposition or "spiralling" of high-tension wires is generally avoided in modern installations. Trouble is more likely to occur with telephone wires. These are preferably carried on a separate set of poles

as far away as possible from the high-tension line; and, whether the power wires are transposed or not, it is well to transpose the telephone wires at frequent intervals—if possible, at every pole. A good quality of insulator should be used for supporting the telephone wires.

CHAPTER VII

POLYPHASE INDUCTION MOTORS

72. THE leading principles underlying the rotation of a short-circuited armature in a rotary magnetic field, produced by two or more alternating currents differing in phase, were explained in article 31, Chapter III., to which the reader is referred, and with which he should make himself thoroughly familiar. A picture of the rotating field, dragging with it the short-circuited armature, is a very useful one; but there is another way of studying the behaviour of the induction motor which will generally be found to have advantages. This consists in considering the induction motor as a slightly modified transformer, the primary coils of which are represented by the stator, or inducing windings, while the secondary coils are movable, being, in fact, the short-circuited windings of the rotor.*

It has been shown (see article 29, Chapter III.) how

* This nomenclature is, of course, based on the assumption that the primary windings—*i.e.*, the coils connected to the supply mains—are at rest, while the short-circuited secondary revolves; but, although this is the most usual arrangement, it must be understood that it is by no means a necessary one. If the primary coils are wound on the revolving portion of the magnetic circuit, then slip rings must be provided for conveying the current to the coils, corresponding in number to the number of phases of the polyphase supply.

a rotary field may be produced by two or more currents having certain phase differences between them; and since two equal currents differing in phase by 90 time-degrees will produce a uniform rotating field, it follows that any uniform rotating field, however produced, can be resolved into two equal alternating components differing in phase by a quarter period; and, instead of studying the problems of induction motors by picturing a magnetic field revolving in space, we shall, in this chapter, treat the subject by imagining two alternating magnetic fields differing in phase by one quarter of a period, and acting together so as to bring about the rotation of the short-circuited armature.

This method is by no means new: indeed, it has always been advocated by Mr. Ll. B. Atkinson, and was used by him so long ago as 1898, in his classic paper on "Alternating-Current Motors," read before the Institution of Civil Engineers.*

Mr. Atkinson, however, in common with many later writers, treats the subject very fully, and even goes into details of design; while the methods and diagrams adopted in the following pages, although not primarily intended for the use of designers, are yet sufficiently accurate for most practical purposes, and lead to some simplifications.

In Chapter V. the theory of the alternating-current transformer was explained on the assumption that magnetic leakage was *nil—i.e.*, that all the magnetism generated in the primary circuit passed also through the secondary coils. But the alternating-current induction motor—owing to the disposition of the windings and the necessary air-gap—is by no means a perfect transformer, and we shall therefore consider, in the first

* *Minutes of Proceedings*, Inst. C. E., vol. cxxxiii., p. 113.

place, what are the effects of magnetic leakage in an ordinary single-phase transformer.

73. **Magnetic Leakage in Transformers.**—Unless the primary and secondary coils of a transformer are wound close together on the same portion of the iron core, there must necessarily be a certain number of magnetic lines generated by the primary current which do not thread their way through all the turns of the

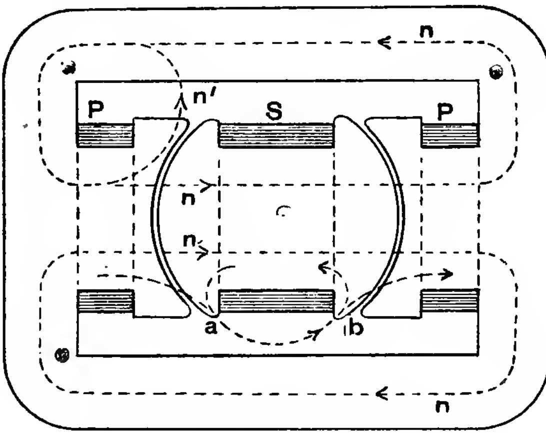

FIG. 78.

secondary coils. The amount of this leakage magnetism will increase with the growth of the primary current, and will, therefore, be approximately proportional to the value of the secondary current.

Fig. 78 is a section through a single-phase transformer in which the magnetic leakage would be considerable. The two primary inducing coils, marked P, are wound upon the iron core at some distance from the

secondary coil, S, and, moreover, an air-gap has been introduced in that portion of the magnetic circuit which lies between the two sets of coils.

When there is no current in the secondary—*i.e.*, when this is open-circuited—nearly all the magnetic lines due to the primary current will pass through the central core, and so thread their way through the secondary coil, because—unless the air-gap be very large—this will be the easier path; and the amount of magnetism finding a shorter path, such as n', will be small. But when current is allowed to flow in the secondary coils, this produces a magnetising force exactly equal and opposite to the magnetising force of the main component of the current in the primary coil. The result is that, although this component of the primary current can produce no magnetic flux through the secondary coil, it will give rise to an appreciable amount of magnetism which will leak across the air spaces and miss the secondary coils.

In treating of magnetic leakage in transformers and induction motors, the assumption is usually made that the secondary winding has self-induction; the argument being as follows:

The current in the primary coils gives rise to a total flux, N, of such a value as to choke back the whole of the applied potential difference which is not lost in ohmic drop. Of this total flux, N_s lines pass through the secondary coils, and N_l lines leak back by other paths. Now, the N_s lines through the secondary induce a certain back E.M.F. which, however, is not in phase with the secondary current, even if the terminals are short-circuited or the outside load is non-inductive. It is assumed that the secondary windings have self-induction, and that the current flowing in them gives rise to another E.M.F., in quadrature with the E.M.F.

due to the flux N_s, thus reducing the useful E.M.F. producing the flow of current.

It is not suggested that this is an incorrect way of treating the subject (in some cases it may be a necessary one); but note that it involves the idea of a certain portion of the total leakage magnetism being *generated* by the current in the secondary coil; and we must, therefore, conceive of two *independent* streams of alternating magnetism (with a phase difference of a quarter period) passing through this coil. In other words, the magnetic flux due to the primary coil is considered as giving rise to a certain E.M.F. in the secondary coil, the current in which produces a back E.M.F. of self-induction, which, together with the above E.M.F. of *mutual* induction, gives us the *resultant* useful E.M.F. in the secondary winding.

The writer's method, which leads to a certain simplification of the diagrams, is based on the assumption that the whole of the magnetic flux which passes through the secondary coil generates an E.M.F. in the secondary windings to which the secondary current is directly due, and that this is the *only E.M.F. generated in these windings.*

Referring again to Fig. 78, it might be objected that the current in the secondary coil, S, is bound to produce leakage magnetism as represented by the dotted line *a b*; but, on consideration, it will be seen that any such magnetic lines may, with equal correctness, be associated with the component of the primary current which balances the secondary current; in which case the closed path of such lines will not be through the central iron core on which the secondary is wound, but through the outside portion of the magnetic circuit. It is unquestionable that the configuration of the magnetic circuit

may be such as to render scientifically inaccurate this method of treating the subject; but, in almost every case which is likely to arise in practice, the error introduced by supposing the secondary to be non-inductive will be negligible. The question practically resolves itself into the correct determination of the core loses in the various portions of the magnetic circuit.

74. **The Induction Motor considered as a Special Case of the Alternate-Current Transformer.**—Fig. 79, which represents a polyphase induction motor in its simplest form, has been specially drawn to illustrate the resemblance between such a motor and an ordinary transformer with a large amount of leakage.

The revolving field is resolved into two equal components with a phase difference of 90 degrees; the two stator coils P P being connected to phase 1, while the two coils O O are connected to phase 2.

In practice, the motor would probably have more than two pairs of poles, and, moreover, the stator or primary windings would be distributed in a number of slots on the inside periphery of the stator ring; but the arrangement shown in the figure is otherwise correct, and will serve the purpose of explanation better than a section showing the actual disposition of the windings in a two-phase motor.

The rotor windings may consist of several equally spaced short-circuited windings; or—if the squirrel-cage type is used—all the conductors, passing through slots on the periphery of the rotor, would be short-circuited by a couple of rings at the ends of the armature. In Fig. 79, each pair of diametrically opposed conductors is supposed to be short-circuited by end connectors, such as *n* and *m*.

The action of the motor may briefly be described as follows:

The coils P P of phase I produce an alternating magnetic field in the direction B B′. This induces no E.M.F. in the rotor coil *n*, of which the axis is at right

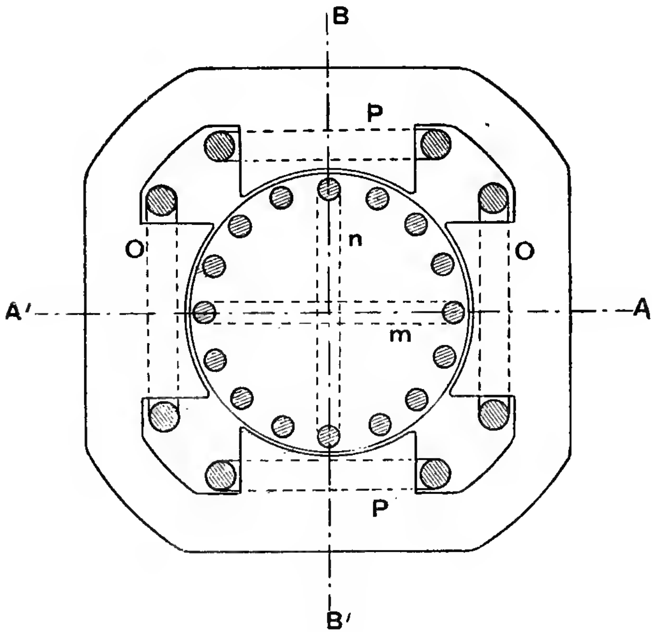

FIG. 79.

angles to the direction of the magnetic flux; but in all the other coils E.M.F.s will be generated by the alternating flux, and, since the rotor windings are assumed non-inductive, the resulting currents will be in phase with the induced E.M.F., and therefore exactly one

quarter period behind the phase of the alternating magnetism. The amount of current flowing in each coil will be proportional to the total flux passing through it, being greatest in the coil *m* lying on the axis A A′.

If the motor is at rest, *no torque will be exerted by these currents reacting upon the field of phase 1*, and the motor will, therefore, not start with only the coils P P excited, even under light load conditions. Observe, however, how the coils O O excited by phase 2 produce a field along the axis A A′ exactly 90 time-degrees out of phase with the field B B′, and, therefore, exactly *in phase with the currents induced by phase 1 in the rotor windings.*

The result is that the magnetism entering the rotor from one set of poles gives rise to currents in the rotor conductors in phase with the magnetic flux from the other set of poles; and since the torque will depend upon the product of the current and field strength—*i.e.*, upon the magnitude of the rotor currents and the intensity of that component of the magnetic field in which the rotor conductors are immersed *which is in phase with the current*—it follows that the motor will be self-starting.

As the speed increases, the E.M.F.s induced in the rotor conductors, due to rotation in the two alternating magnetic fields, will tend to reduce the amount of the rotor currents; and the maximum speed will be attained when the E.M.F.s generated in the conductors, as they cut through the magnetic flux, almost exactly equal and balance the E.M.F.s due to induction—the difference being such as to allow the very small current to pass which is necessary to overcome bearing friction, iron loss, windage, etc.

The maximum possible speed, if there were no losses in the motor, would be the speed of synchronism; and,

since the percentage losses, when running light, are very small, an unloaded induction motor may be said to run practically at synchronous speed. In order to illustrate and amplify this statement, it should be mentioned that by synchronous speed is meant a speed of rotation equal to that of the revolving field. If the primary coils of a machine are so wound as to produce only two poles per phase, as in Fig. 79, the magnetic field will revolve at the rate of f revolutions per second, all as explained in Chapter III., article 29. If there are four poles per phase, the rotating field will make only $f/2$ revolutions per second, and so on, the synchronous speed being expressed by the ratio f/p, where p is the number of *pairs* of poles per phase, and f is the frequency.

Consider a rotor coil of S turns, the "pitch" of which is the angular distance between consecutive poles of opposite sign on one phase. The average value of the E.M.F. *induced* in such a coil by the alternating flux is

$$4\ \mathrm{N}f\mathrm{S},$$

and the corresponding value of the E.M.F. *due to rotation* is

$$n \times 2p \times \mathrm{N} \times 2\,\mathrm{S},$$

where n = revolutions per second, and N = maximum value of the flux per pole. If, now, we equate these two quantities to obtain the condition of maximum possible speed, we get

$$n = f/p,$$

which confirms the earlier statement that the maximum possible speed—reached only if the losses can be considered negligible—is the speed of synchronism.

Thus, if the motor be supposed to have only one pair

of poles per phase, as in Fig. 79, and if the frequency of the stator current be 25, then the number of revolutions per minute when running light will be 25 × 60, or 1,500, because this is the speed which will make each conductor cut the magnetic flux of any one phase in both directions during the time of one complete cycle of magnetisation.

75. **Vector Diagram for Induction Motor with Rotor at Rest.**—The condition of things at the moment of switching on the stator current, while the rotor is still at rest, is represented by the diagram Fig. 80.

The construction of this diagram is generally similar to that of Figs. 51 and 52 in Chapter V.; but whereas these refer to a transformer supposed to be without magnetic leakage, Fig. 80 refers to a transformer—such as an induction motor—in which magnetic leakage plays an important part.

If the rotor is of the squirrel-cage type, the condition is that of a transformer having appreciable magnetic leakage and a short-circuited secondary. If, on the other hand, the motor under consideration has a wound rotor with resistances connected in series (for reasons which will be explained later), then the condition is merely that of a transformer with its secondary closed on a definite non-inductive load. The diagram Fig. 80 applies to either case, and, for the purpose of simplifying the construction, we shall assume (as when treating of static transformers) that the ratio of primary to secondary turns is 1 : 1.

It must, moreover, be clearly understood that the diagram represents the relation of E.M.F. and current in only *one of the phases*, such as that which excites the coils P P (Fig. 79); but exactly the same relation will hold good in the other phase (coils Q Q, Fig. 79), with

the one exception that the diagram, as a whole, must be supposed to be turned through an angle of 90 degrees.

The construction of the vector diagram Fig. 80 is as follows:

Draw the vertical line, B′ B, to represent the phase of that component of the total magnetic flux which passes

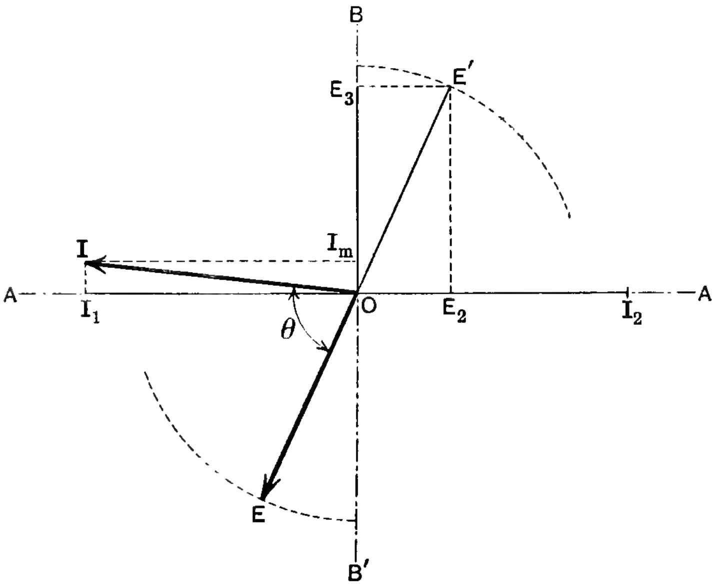

FIG. 80.

through the rotor. This will induce an E.M.F. O E_2 in the rotor windings, lagging exactly a quarter period behind the magnetism, O B; the magnetising component of the primary current being O I_m, in phase with the magnetism. The rotor current O I_2 will be in phase with E_2, and equal in magnitude to $E_2 \div R$, where R is

the resistance of the rotor windings. (It must not be forgotten that the secondary or rotor windings are supposed to be without self-induction.)

This secondary current must be balanced by a primary current component, O I_1, exactly equal but opposite to O I_2; the total primary current being O I, obtained by compounding O I_1 and O I_m.*

Of the two components of this total primary current, it is the "wattless" component, O I_m, which produces the flux through the rotor, while the component O I_1 gives rise to the leakage flux which, although indirectly due to the secondary current, does not pass through the rotor. This leakage flux, being in phase with O I_1, produces an E.M.F. component O E_3 in the primary windings only, exactly one quarter of a period behind O I_1.

By combining this E.M.F. with the induced E.M.F., O E_2—due to that portion of the total flux which passes through the rotor—we obtain the vector O E′, which is the total back E.M.F. in the primary circuit. If we assume the resistance of the primary coils to be negligible, the impressed primary E.M.F. will be O E, exactly equal and opposite to O E′; the power factor, for this particular

* We are neglecting the losses in the motor, which would make O I slightly greater than as obtained by this construction; but, in any case, the "energy" component of the total magnetising current is very small in a motor in comparison with the "wattless" component, O I_m. This is due to the necessary air-gap; and, whereas in a well-designed static transformer the true magnetising current might be about 45 degrees in advance of the magnetic flux (see Figs. 51 and 52) in a motor, it would be much more nearly in phase with the magnetism. Thus, when the motor is running light, at synchronous speed, the "energy" component of the no-load primary current — required to overcome frictional, windage, hysteresis, and eddy-current losses—would not exceed about 15 per cent. of the "wattless" component, O I_m, which is in phase with the magnetism.

condition of the rotor at rest, being cos θ, where θ is the angle between the vectors O I and O E.

76. **Starting Torque of Polyphase Induction Motor.**—Referring still to the diagram Fig. 80, we see that the rotor current I_2 is exactly a quarter period out of phase with the magnetic induction, O B, which enters the rotor. The rotor current cannot, therefore, react upon this field so as to produce a starting torque; and if one phase only of a polyphase motor is connected to the supply, the motor will not be self-starting. But, as previously pointed out, the other set of coils (O O, Fig. 79) produces a magnetic flux through the rotor, equal in amount, but exactly 90 degrees out of phase with the flux, B B′, due to the coils P P. This flux, being in the direction A A′ (Fig. 80), *is exactly in phase with the current* O I_2 *in the rotor*, as already explained on p. 200.

The starting torque will, therefore, be proportional to *rotor current* × *magnetic flux entering rotor*, or, since O E_2 is—to a certain scale—a measure of the magnetic flux component which passes through the rotor, we may write

$$\text{starting torque} \propto \text{O } I_2 \times \text{O } E_2.$$

In Fig. 80, if the impressed stator E.M.F. is supposed constant, the ends of the vectors O E′ and O E must necessarily move on the dotted circle described from O as a centre, and it will be an easy matter to study the effect, upon the starting torque, of varying the resistance of the rotor windings, while maintaining constant the supply pressure at the stator terminals.

As regards the running condition, it is evidently an advantage to keep down the resistance of the rotor conductors in order to avoid large I^2 R losses: but note, at the outset, that *if the rotor conductors were entirely without resistance, the starting torque would be nil.* The smallest

amount of magnetic flux passing through the rotor would give rise to an enormous current in the rotor conductors; the result being that the current I_2 would immediately rise to such a value as to cause the whole of the primary

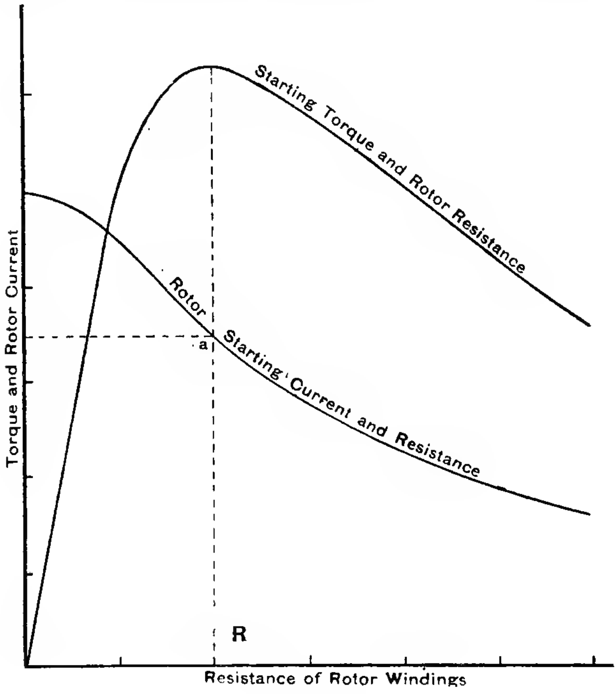

FIG. 81.

flux to be leakage magnetism, in the phase O A, thus bringing O E′ to coincide with O B, while O E_2 would be equal to zero.

The curves of Fig. 81 have been plotted from measure-

ments taken off Fig. 80: the upper curve shows how the starting torque reaches a maximum for a definite resistance, R, of the rotor windings, after which, for any further increase of resistance, the torque decreases rapidly. This particular value of the resistance is such as to cause the leakage magnetic flux to be equal to the useful flux which passes through the rotor. In other words, it is the rotor resistance which will make E_3, in Fig. 80, equal to E_2, thus causing the product $E_2 \times E_3$ to be a maximum. The reason of this is not far to seek, for the induced leakage E.M.F., $O\ E_3$, may be taken as being proportional to the current producing it (seeing that the path of the leakage magnetism is largely through air, which is of constant permeability). The length of the vector $O\ E_3$ is, therefore, a certain multiple of the length $O\ I_2$, and, instead of writing

$$\text{torque} \propto O\ E_2 \times O\ I_2,$$

we may write

$$\text{torque} \propto O\ E_2 \times O\ E_3.$$

With regard to the resistance, R, this is evidently equal to $O\ E_2 \div O\ I_2$, which can also be written

$$\text{rotor resistance} \propto O\ E_2 \div O\ E_3.$$

The upper curve in Fig. 81 is obtained by plotting the product $(O\ E_2 \times O\ E_3)$ with the corresponding ratio $\frac{O\ E_2}{O\ E_3}$; the resultant, $O\ E'$ (or $O\ E$, the impressed volts), being supposed of constant value.

The lower curve shows the relation between current and resistance; it has been obtained by plotting $\frac{O\ E_2}{O\ E_3}$ as abscissæ, with the corresponding values of $O\ E_3$ as ordinates.

It will be understood that both curves in Fig. 81 will

continue to approach zero value as the resistance of the rotor is increased. When the rotor windings are open-circuited—*i.e.*, when R = infinity—the torque will evidently be zero, since, although the magnetic field through the rotor will have reached its maximum value, there will be no current to react upon it and produce a torque.

77. **Vector Diagrams for Induction Motor with Armature in Motion.**—When the starting torque, acting upon the armature or rotor, causes the same to revolve, another E.M.F. is generated in the rotor windings. This is the E.M.F. of rotation, due to the cutting of the alternating magnetic field by the rotor conductors: its value will depend upon the amount of the magnetic flux cut by the revolving conductors, and the velocity at which they cut through this field; it will, moreover, be in phase with the magnetism. The vector diagram, as drawn in Fig. 80, must, therefore, be modified slightly, and it will now be as drawn in Fig. 82, which applies to the condition of a motor running up to speed.

Let us suppose that the current in rotor—which will be *less* than at starting, on account of the back E.M.F. of rotation—has been reduced to the value represented by the length of the vector O I_2. The corresponding component of the primary current will be O I_1, which produces the leakage field in the direction O A. This gives rise to the leakage induced E.M.F. O E_3, which will be approximately proportional to O I_1, and, therefore, of a smaller value than in the diagram Fig. 80. This enables us to obtain the vector O E_2 as representing the induced E.M.F. in rotor windings—the magnitude of the resultant E.M.F. vector, O E″, remaining as before.

The magnetising current, O I_m, which has to produce an increased flux through the rotor, will be *greater* than in Fig. 80; it will be approximately proportional to

O E_2. The total primary current will be O I, and the impressed E.M.F., O E—drawn equal and opposite to O E″, since we are neglecting the I R drop in primary.

Now mark off O E_r, in the same phase as O I_2, to represent the drop of pressure in rotor windings. (The length O E_r will bear the same proportion to O I_2 in Fig. 82 as O E_2 bears to O I_2 in Fig. 80.) This leaves E_r E_2 to be balanced by an exactly equal E.M.F. of

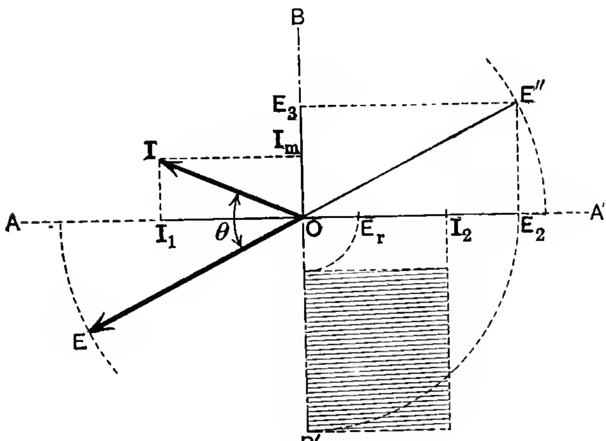

FIG. 82.

rotation *due to the conductors cutting through the magnetic lines which enter the rotor a quarter period in advance of the flux* O B.

The action, when running, may be summed up as follows:

Phase B of the simple two-phase motor indicated by Fig. 79—when connected to the field coils, P P—produces a magnetic flux in the direction B′ B, through the

rotor. This gives rise to currents in the rotor coils, *which are in phase with the magnetic flux due to the magnetising current of phase* A *in the field coils* O O. When the armature begins to revolve, a back E.M.F. is produced in these rotor coils, exactly opposite in phase to the current already flowing. It tends, therefore, to reduce this current, and actually does so, until the difference between the induced E.M.F. E_2 (Fig. 82), due to the magnetism of phase B, and the back E.M.F. due to rotation in the magnetic field of phase A, leaves a resultant or effective E.M.F., E_r, just sufficient to send such a current through the rotor as will give the necessary torque.

It will be understood that exactly the same thing occurs a quarter period later, when the same coils that occupied a horizontal position such as *m* (Fig. 79) now occupy a vertical position such as the coil *n*, only in this case the field due to phase A, exciting the coils O O, gives rise to currents in the rotor coils, while the back E.M.F. of rotation in these coils is due to their cutting through the magnetic flux from the poles, P P. The arrangement, in fact, is symmetrical, and each set of stator coils gives rise to currents in the rotor which react upon, and are in turn influenced by, the magnetic flux generated by the other set of coils.

In Fig. 83 the diagram has been drawn for the condition of practically synchronous speed—*i.e.*, motor running light—the only work done being that required to overcome the I^2 R losses in the rotor.*

The whole of the induced E.M.F. will now be in the direction O A′, and O E_2 will be equal in length to the vector of the impressed volts O E, since the leakage

* For simplicity of construction, the primary losses are still considered negligible.

flux due to the very small rotor current, O I_2, may be neglected. The magnetising current, O I_m, will have reached its maximum value, and the total primary current will be O I, only very slightly in advance of the magnetising component, O I_m, owing to the comparatively small value of O I_1. In a well-designed motor, the total primary current, O I, when running light, might be about one quarter of the full-load current.

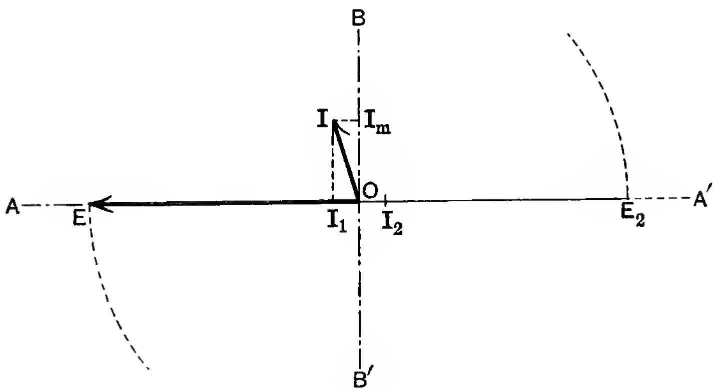

FIG. 83.

Reverting again to Fig. 82, this diagram, if studied attentively, will prove very instructive. It will serve to show that the polyphase motor may be compared with advantage to a direct-current shunt-wound motor, the behaviour of these two types of machines—both when starting up and when running under load—being in many respects similar.

The torque exerted by the armature, as explained in article 76, is proportional to O $E_2 \times$ O I_2; and since

$O\ E_3 \propto O\ I_2$, the area of the dotted rectangle $O\ E_2\ E''\ E_3$ is a measure of the torque.

Again, the E.M.F. of rotation, represented by the length $E_2\ E_r$, will depend upon the amount of the flux entering the rotor, and the rate of cutting, or speed of the motor; and since the former quantity is represented to a certain scale by the length $O\ E_2$, we may write

$$\text{length } E_2\ E_r \propto O\ E_2 \times \text{speed},$$

or,
$$\text{speed} \propto \frac{E_2\ E_r}{O\ E_2}.$$

With regard to the power given out by the motor, this is the product of torque and speed, or

$$\begin{aligned}\text{power} &\propto \text{torque} \times \text{speed} \\ &= (O\ E_2 \times O\ I_2) \times \frac{E_2\ E_r}{O\ E_2} \\ &= O\ I_2 \times E_2\ E_r,\end{aligned}$$

which is a measure of the power, expressed in watts per phase.

This is graphically represented by the shaded rectangle (Fig. 82), and, since the conditions which occur in phase A are repeated in phase B, *twice* the watts obtained by the above construction will be the total power developed by the rotor.

It is important to know what is the amount of overload that an induction motor will stand—apart from considerations of heating or efficiency—before it will cease to respond to a further increase of load; and this point, together with the reasons for inserting resistances in rotor windings for starting purposes, will be dealt with in the following article.

78. **Overload Capacity of Induction Motor as influenced by Magnetic Leakage and Rotor Resistance.**—Apart from considerations of mechanical strength and excessive temperatures, the question arises as to what are the output limitations of the induction type of motor on momentary overloads.

The output is the product *torque* × *speed*; but as the speed of an induction motor varies little between no load and full load conditions, we shall consider, in the first place, the production of a *maximum torque* independently of the speed. It has already been shown that

$$\textit{torque} \propto \textit{secondary current } I_2 \times \textit{secondary induced volts } E_2,$$

because the last term is a measure of the magnetic flux entering the rotor. It was also shown that the vector $O\,E_3$ (Fig. 82), representing the counter E.M.F. in the primary coils due to the leakage flux, is approximately proportional to the current component I_1, and therefore to I_2. Thus, in terms of the quantities in Fig. 82,

$$\textit{torque} \propto O\,E_2 \times O\,E_3,$$

and this product will have a maximum value when $O\,E_2 = O\,E_3 = \sqrt{2}\,O\,E$. This leads to the conclusion that the maximum torque will be obtained when the secondary current has such a value that the leakage flux is equal to the useful flux. Also, since the induced pressure (E_2), which corresponds to the condition of maximum torque, is a particular multiple of the impressed voltage (E), we may write,

$$\textit{maximum torque} \propto I'_2 \times E,$$

and, for a constant supply voltage,

$$\textit{maximum torque} \propto I'_2,$$

where I'_2 is the particular value of the rotor current which makes the leakage flux equal to the useful flux. Obviously, the smaller the air-gap between stator and rotor, and the more carefully the machine is designed with a view to keeping down the leakage magnetism, the *greater* will be the required value of I_2 to produce the condition of maximum torque. It is the magnetic leakage due to the rotor current—or, more correctly speaking, to its balancing component in the primary circuit—which determines the maximum value of this current, beyond which any further increase will only lead to a reduced torque; and since the motor will be incapable of responding to a further addition to the external load, it will, at this point, slow down and come to rest.

It is interesting to note that the value of the rotor coil *resistance* does not enter into these considerations of maximum *torque*; but this resistance, since it involves the idea of power loss, must necessarily enter into the determination of the maximum *output*.

Referring again to Fig. 82, it is evident that the length $O\,E_2$ will be constant for a given value of the vector $O\,I_2$; but the portion $O\,E_r$ of this total induced E.M.F. in the rotor will depend upon the resistance of the rotor windings: it is—as already explained—the E.M.F. required to overcome the resistance, and will, therefore, be equal to $I_2 \times R$, where R stands for the rotor resistance.

Hence, for a given secondary current, such as the critical value corresponding to the condition of maximum torque, the length $O\,E_r$ will be proportional to the rotor resistance. But it is also proportional to the *slip*, because —as explained on p. 212—the rotor speed may be expressed by the ratio $\frac{E_2\,E_r}{O\,E_2}$, which is the same thing

as stating that the length $E_2 E_r$ is a measure of the actual speed, to the same scale as $O E_2$ represents the maximum or synchronous speed; thus leaving the *difference* $(O E_2 - E_2 E_r)$ or $O E_r$, to stand for the *slip revolutions*, or *relative speed of revolving field and rotor.*

The conclusion to be drawn from the foregoing arguments is that—although the maximum torque exerted by a motor is not affected by the resistance of the rotor winding—the drop in speed which must occur before the maximum torque is reached—*i.e.*, before the machine will cease to respond to a further increase of load—depends upon the rotor resistance, and, indeed, this *drop in speed*, or *percentage slip*, is *directly proportional to the resistance of the rotor windings.*

Example.—If the particular value of the rotor current producing maximum torque is 100 amperes per phase, and the induced E.M.F. (E_2) per phase is 12 volts, a rotor resistance of $\frac{12}{100} = \cdot 12$ ohm would result in the maximum torque being exerted at the moment of starting—*i.e.*, when the speed = 0, or slip = 100 per cent. But if we reduce this resistance to ·06 ohm, the maximum torque will be exerted when the speed is one-half synchronous speed (slip = 50 per cent.). If the resistance is only one-sixth of the above value—*i.e.*, ·02 ohm—the breakdown speed would be five-sixths of synchronous speed (slip = one-sixth).

The curves of Fig. 84 have been drawn to illustrate this point.

Curve A would apply to a motor without any external resistance in rotor; it refers to a machine which would break down when the slip reaches a value equal to 20 per cent. of synchronous speed, or five times what we have supposed to be the slip at normal full load (4 per

cent.): the maximum torque being—in this example—a little over three times the full-load torque.

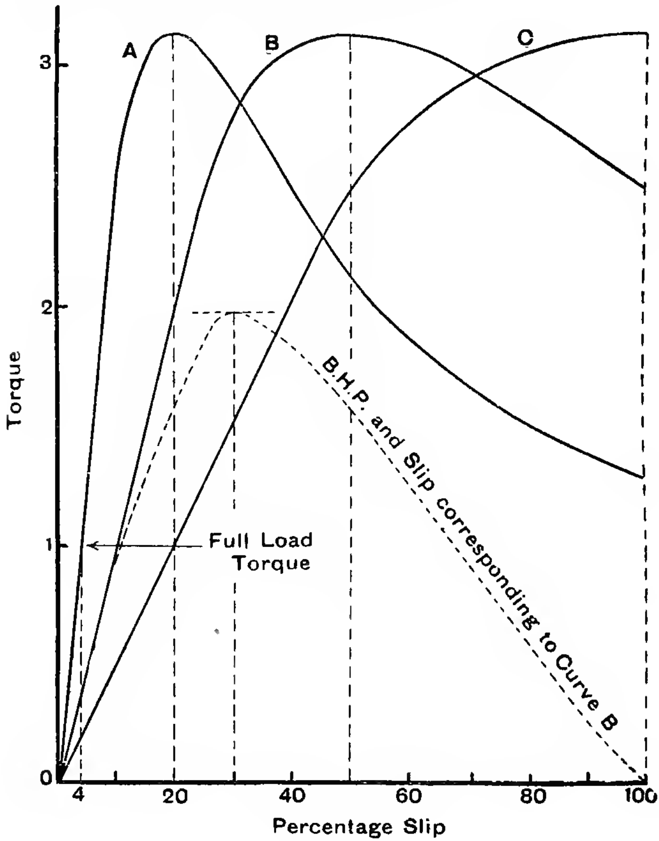

FIG. 84.

If the resistance of the rotor windings is increased in the proportion of 5 to 2, the maximum torque will occur

at half the synchronous speed (slip = 50 per cent.). (See curve B.)

By still further increasing the rotor resistance, until it is five times as great as the value corresponding to curve A, we obtain curve C, which shows how the torque would increase with the slip until the maximum value is reached with the rotor at rest (slip = 100 per cent.).

It should be noted that, for small values of the slip, the curves A, B, and C are practically straight lines, and, for this reason, it is always permissible to assume that the slip is directly proportional to the torque for all values from zero up to the rated full load of an induction motor.

Although it is the maximum *torque* which determines the speed at which the motor will break down—*i.e.*, fail to provide the increased turning effort demanded by a further increase of load—it does not follow that this particular speed corresponds with the greatest possible *output* or brake horse-power. The maximum output would occur at a somewhat smaller slip than that which corresponds with the breaking-down point; because, although the *torque* may continue to increase, the *speed* is being reduced, and the maximum value of the product *speed* × *torque* will occur before the torque has reached its maximum possible value.

This is clearly shown by the dotted curve in Fig. 84, which has been plotted to show the relation between speed and power for the particular rotor resistance corresponding to the curve B. It will be seen that the maximum value of this dotted curve occurs with a speed of 75 per cent. (slip = 25 per cent.), whereas the maximum value of the torque curve B occurs at a speed of 50 per cent. of synchronism (slip = 50 per cent.).*

* The overload capacity of a polyphase motor just before falling out of step would, in practice, be something between twice to three times or even four times the rated maximum load. The values of

79. **General Conclusions regarding Magnetic Leakage in Induction Motors.**—The objections to the leakage magnetism, which is produced by the stator coils, but fails to pass through the rotor, are fairly evident. This dissipation of magnetic flux, which cannot be utilised, leads to a reduction in the overload capacity of the motor, and a diminished torque, for a given slip, at intermediate loads. It also reduces the power factor, and even the efficiency, since it involves increased hysteresis and eddy-current losses in the iron cores and neighbouring metal work.

The primary magnetising current is usually between 25 and 35 per cent. of the normal full-load current, and it should clearly be the aim of the designer to keep it as low as possible.

The chief remedy for leakage troubles in alternating-current motors, and other alternating-current apparatus, is to work with currents of low periodicity;* this permits of a reduction in the necessary number of poles in the motor for a given speed of rotation, and also allows of *higher inductions being used in the iron cores.* This is a distinct gain; and in many cases where improvements in the working of alternating-current apparatus are attributed to the adoption of reduced frequencies, the improvements are not to be accounted for by the reduction of frequency *per se*, but to the fact that the induction in the iron cores has been increased.

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slip corresponding to curves B and C of Fig. 84 are larger than would occur in a commercial machine: the slip at full load rarely exceeds 4 per cent. of synchronous speed.

* The expense of working at very low frequencies is, however, great; especially when static transformers are used—as is very often the case—for raising and lowering the pressure at both ends of a transmission line.

The following considerations will make it clear that, with a view to keeping down the percentage leakage loss, it is advantageous to make the magnetic induction in the iron as high as possible; although it must not be overlooked that, for a given frequency, the practical limit in this direction is set by the amount of the hysteresis and eddy-current losses, which increase largely with the higher inductions: but by keeping down the frequency, higher inductions are permissible.

Let B_s stand for the maximum value of that component of the total induction in primary core which passes also through the secondary or rotor windings, and B_l for the maximum value of the leakage component of the total induction in primary core.

Then, assuming the reluctance of the path of the leakage lines to be constant, we may write

$$B_l \propto \text{ampere-turns tending to produce leakage,}$$
$$\propto T \times I_1,$$

where T = the number of turns in the primary or stator winding, and I_1 = the main component of the primary current. And since the mean back E.M.F. in primary, due to leakage (the vector O E_3 in the last few diagrams), is proportional to $B_l \times T \times f$, where f = the frequency, it follows that we may write

$$E_3 \propto T^2 I_1 f,$$

from which we see that the mean value of the "leakage volts" for any given arrangement of the magnetic circuit is proportional to the square of the number of turns in the winding, and to the current, and to the frequency.

But as it is not the actual value of the leakage E.M.F. with which we are now concerned, it will be advisable to express this as a percentage of the E.M.F. available

for generating current in the secondary circuit. This last will be proportional to $B_s \times T \times f$; and if we divide the actual "leakage E.M.F." by this amount, we obtain the expression:

Percentage back E.M.F. due to leakage

$$\propto \frac{T^2 I_1 f}{B_s T f}$$

$$\propto \frac{T I_1}{B_s}.$$

It follows that, if we consider a transformer under given load conditions, the *percentage* back E.M.F. due to leakage is independent of the frequency, but is inversely proportional to the induction in the core enclosed by the secondary winding; and since the induction motor may be regarded as a transformer with short-circuited secondary, the above reasoning is applicable to this special form of alternating-current transformer.

80. **Complete Vector Diagram for Polyphase Induction Motor.**—Up to the present we have supposed the stator or primary winding to be of negligible ohmic resistance, and it will, therefore, be advisable to examine briefly how the problem is influenced by taking this resistance into account.

Fig. 85 is the complete diagram for a polyphase induction motor. It differs in only two respects from the diagram Fig. 82. In the first place, the magnetising current component, O I_m, has been drawn rather more than 90 degrees in advance of the induced E.M.F., E_2, so as to take into account the small "energy" component of the total primary current, required to provide for the hysteresis and eddy-current losses. In the second place, another component O E'_r, of the primary impressed

E.M.F. has been taken into account. This is the component required to overcome the resistance of the stator windings.

The construction of the diagram Fig. 85 is exactly similar to that of Fig. 82 up to the point of obtaining the primary vector O E_1 (corresponding to O E of Fig. 82); but we have now to add another component O E'_r—

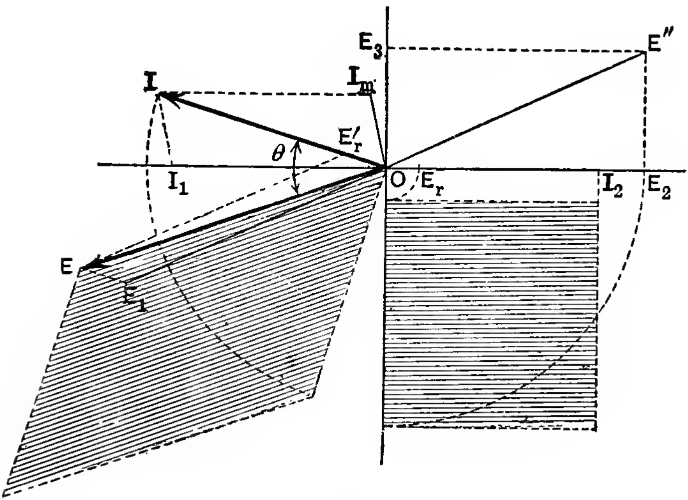

FIG. 85.

equal to I × R, where I = the total primary current, and R = the primary resistance—in order to obtain the required potential difference, O E, at stator terminals. The introduction of this component of the primary E.M.F. leads to a reduction of the angle θ, and, therefore, an improvement in the power factor; but there is evidently no advantage in increasing the power factor at the cost of additional *wasted* energy. A high primary

resistance cannot be otherwise than objectionable; it absorbs a certain amount of energy which would otherwise have been available for doing useful work.

81. **Efficiency of Polyphase Motors.**—The two shaded parallelograms in Fig. 85 represent respectively the power which is supplied to the primary terminals, and that which is transmitted to the shaft by the revolving rotor.

The power, per phase, supplied to the primary terminals is $E \times I \times \cos\theta$, and it is proportional to the area of the parallelogram constructed on one of the vectors, O E, with the other vector, O I, moved round through an angle of 90 degrees. The power utilised in driving the shaft is $I_2 \times E_2$ *less* the $I^2 R$ losses in the rotor windings, or $I_2 \times (E_2 - E_r)$, and, if windage and friction losses are neglected, the *efficiency* will be the ratio of this last quantity to the power put into the motor at primary terminals.

The full-load efficiency of commercial polyphase motors will vary between 75 per cent. for small sizes and 93 per cent. for large sizes; and it will fall off somewhat on an appreciable overload.

The following table, compiled from figures referring to a large number of three-phase motors of various makes, may be of use as indicating the average full-load efficiency and power factor likely to be obtained with this class of machine:

B.H.P.	Efficiency.	Power Factor, Full Load.
2	·76	·81
5	·81	·84
10	·85	·90
20	·88	·91
50	·91	·92
200	·93	·93

82. **Methods of Measuring the Slip of Induction Motors.**—One of the most important factors to be measured when testing an induction motor is the slip. A large slip for a given load is an indication of large losses in the rotor windings. We have already seen how the slip, for a given torque, is proportional to the resistance of the rotor windings. It has also been explained in connection with the vector diagrams (see, for instance, Fig. 85) that the slip, expressed as a fraction of the synchronous speed, is $\frac{E_r}{E_2}$, where E_r stands for the volts required to overcome rotor resistance, and E_2 is the E.M.F. induced in the rotor (the difference, $E_2 - E_r$, being the back E.M.F. due to rotation in the magnetic field).

Now, it is evidently equally correct to write

$$\textit{percentage slip} = 100 \times \frac{I_2 \times E_r}{I_2 \times E_2},$$

which amounts to defining the slip as the ratio of rotor copper losses to the total power imparted to the rotor conductors by induction from the primary circuit.

If, therefore, we know the rotor resistance and current, we can calculate the slip. If the rotor is of the squirrel-cage type, the equivalent resistance can be ascertained as follows:

Clamp the rotor so as to prevent rotation: supply a low pressure to the stator terminals until the required primary current is obtained: measure the total power supplied to the primary, and deduct the calculated I^2 R losses in stator windings. This leaves a quantity representing with sufficient accuracy the I^2 R losses in the rotor, corresponding to a definite primary current.

Then, if W is the *output*, expressed in watts, the *slip* is

$$S = \frac{I^2 R}{W + I^2 R}.$$

Evidently the slip could be measured by taking careful readings of the speed when running light (synchronous speed), and, again, when running under load. The

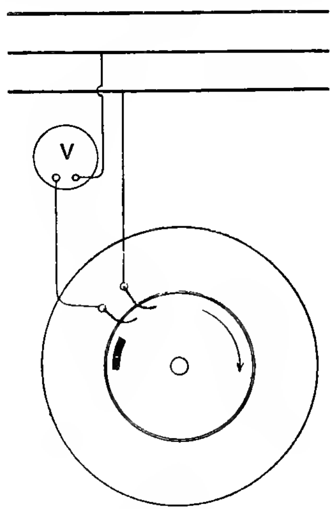

FIG. 86.

difference would be the slip: but as this is a very small quantity relatively to the two speed measurements, the inaccuracy and disadvantages of such a method are obvious.

A number of satisfactory methods of measuring the slip will suggest themselves to anyone with a little ingenuity who cares to study the question. Two useful methods will be briefly referred to.

Imagine a small contact stud fixed to any convenient point on the rotor, in such a position as momentarily to close an electric circuit once during each revolution. This circuit should be connected, through a voltmeter, to the same source of supply as the stator windings (see Fig. 86). If the motor were running at synchronous speed, the indication of the voltmeter would be steadily of the same value, since the local circuit would always be closed at exactly the same instant in the cycle of the supply pressure. But if the machine is running at something less than synchronous speed, the pointer of the voltmeter will take up a swinging motion. This is due to the local circuit being closed each consecutive time slightly later in the E.M.F. cycle; and every double oscillation of the pointer will correspond to one complete wave of alternating E.M.F., thus indicating that—during the time of one such double swing—the rotor is behind the position it would have occupied if synchronous, by twice the angle subtended by two consecutive poles of the same phase. By counting the number of double swings per second, and dividing this by the (known) frequency of supply, we get the percentage slip.

Methods based on this principle have given very satisfactory results. Sometimes a telephone receiver is used in place of the voltmeter, with advantage.

Another simple method of measuring the slip consists in fixing to the shaft of the motor a circular disc having equally spaced black and white segments painted upon it, the number of each kind corresponding to the number of pairs of poles per phase in the stator.

If this is illuminated by an arc lamp connected to the same source of supply as the stator, the disc will appear to revolve backward; and the number of apparent revolutions in a given time will be a measure of the slip.

Thus, if p = the number of pairs of poles per phase, and f = the frequency, and if the number of apparent revolutions be counted during a time equal to $100 \times \frac{p}{f}$ seconds, this number would represent the percentage slip.

As an example, suppose $p = 8$ and $f = 40$. Now count the number of apparent revolutions of the disc during a time equal to

$$1{,}000 \times \frac{8}{40} = 200 \text{ seconds}$$
$$= 3 \text{ minutes } 20 \text{ seconds};$$

let us say that 38 apparent revolutions are observed in this time; then the slip corresponding to the particular load conditions under which the observation was made, would be 3·8 per cent.*

83. **Circle Diagrams.**—Although the vector diagrams previously used are useful for explaining the principles underlying the behaviour of induction motors, the construction for obtaining the primary current and power factor for various values of the rotor current is lengthy, and, moreover, in the case of Fig 85, where the primary resistance is taken into account, a correction has to be made in order to reduce all results to a constant value of the impressed volts, E. It is true that this correction merely amounts to a simple proportion sum, determining the *scale* by which the various vectors should be measured; but, as will be seen, the circle diagrams (originally introduced by Mr. Alex. Heyland) have much to recommend them, and are very convenient to use.

* For a description of an ingenious direct-reading indicator on the above principle, see article by Mr. C. V. Drysdale in the *Electrician* of August 25, 1905.

In its simplest form, the circle diagram for an induction motor is shown in Fig. 88. This applies to a motor in which all I^2 R losses in the copper, and hysteresis and eddy-current losses in the iron, are supposed to be so small as to be negligible.

The corresponding vector diagram, in the form with which we are already familiar, has been drawn in Fig. 87.

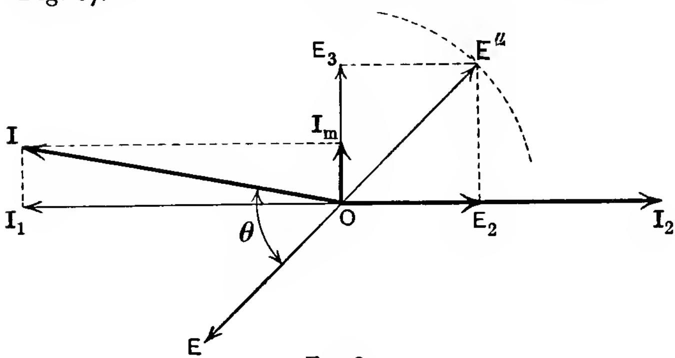

FIG. 87.

It will be seen that Fig. 87 is practically identical with Fig. 82 (p. 209), but it has been re-drawn so as to facilitate direct comparison with the circle diagram.

There is a definite relation between the length of the rotor current vector, O I_2, and the magnetising current component, O I_m, which may be deduced as follows:

The secondary E.M.F., or volts induced in the rotor, will be proportional to the amount of the magnetising current,* or

$$E_2 = I_m \times K_m \qquad (1)$$

* The magnetic circuit is supposed to be of constant permeability, and, therefore, the amount of the magnetic induction is propor-

where K_m is a constant. Moreover, the leakage volts, E_3, are proportional to the rotor current, I_2, or

$$E_3 = I_2 \times K_2. \qquad (2)$$

where K_2 is another constant.

A third condition is that

$$E_2^2 + E_3^2 = E^2, \qquad (3)$$

where E stands for the impressed voltage, and is, therefore, a constant.

Inserting values for E_2 and E_3 from equations (1) and (2), we get,

$$I^2_m\, K^2_m + I_2^2\, K_2^2 = E^2. \qquad (4)$$

There is obviously another equation, which gives the total primary current in terms of its components, and this is,

$$I^2 = I^2_m + I_2^2, \qquad (5)$$

because the triangle O I I_m of Fig. 87 is right angled, with O I as the hypotenuse.

Consider, now, the circle diagram Fig. 88. Draw the vertical line O E to represent the phase of applied E.M.F., and on O B—at right angles to O E—make O A equal to the maximum possible value of the magnetising current, and O B equal to the maximum possible value of the rotor current. These quantities are expressed in terms of the supply voltage and the assumed constants thus:

---

tional to the current producing it. This assumption is justified on account of the comparatively low inductions used in the iron, and the relative importance of the air-gap in the total reluctance of the magnetic circuit.

By (1) $I_m = E_2 \div K_m$

and *maximum* value of $I_m = O\,A = E \div K_m$;

also by (2) $I_2 = E_3 \div K_2$

and *maximum* value of $I_2 = O\,B = E \div K_2$.

On A B and O A as diameters, describe the semicircles as shown. Then, for all intermediate values of the primary current, the end, P, of the primary current vector, O P, will lie on the larger circle. The

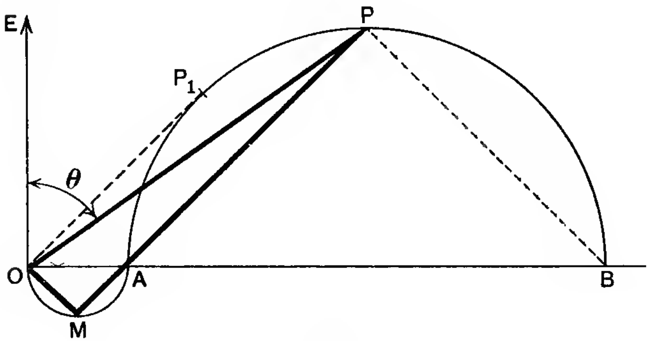

FIG. 88.

line P M represents the rotor current, while O M is a measure of the magnetising component of the total primary current, O P. The angle E O P is the angle of lag, the cosine of which is the power factor; and the phase relations of the other two current components are also correctly shown relatively to the pressure vector, O E.

This diagram correctly reproduces the conditions of Fig. 87, although in a somewhat different form. That the same relations still hold between the various current vectors can be proved as follows:

In the first place, the triangle O P M is always right angled, with O P as the hypotenuse, and this is all that is needed to satisfy equation (5). In regard to the other relations between current components, the triangle O M A is always right angled whatever may be the position of the point P on the larger semicircle. Thus,

$$(\mathrm{O\,M})^2 = (\mathrm{O\,A})^2 - (\mathrm{A\,M})^2,$$

but A M is a definite fraction of P M, in the proportion of O A to O B; thus,

$$\mathrm{A\,M} = \mathrm{P\,M} \times \frac{\mathrm{O\,A}}{\mathrm{O\,B}}$$

$$= \mathrm{I}_2 \times \frac{\mathrm{K}_2}{\mathrm{K}_m},$$

and inserting for O A its previously ascertained value in terms of the impressed voltage, we get,

$$(\mathrm{O\,M})^2 = \frac{\mathrm{E}^2}{\mathrm{K}_m{}^2} - \frac{\mathrm{I}_2{}^2\,\mathrm{K}_2{}^2}{\mathrm{K}^2{}_m},$$

which value for the square of the magnetising current is the same as that given by equation (4). No further proof should be necessary to establish the complete similarity between the circle diagram Fig. 88 and the fundamental vector diagram Fig. 87.

It is interesting to note that the maximum possible power factor—*i.e.*, the smallest angle θ—occurs when the primary current vector is tangential to the large semicircle, as indicated by the dotted line, O P_1, and this shows very clearly the bad effect, as regards the power factor, of a large air-gap and resulting magnetising current, O A, and of bad design generally, leading to a large leakage field, or increased ratio of O A to O B. If a larger air-gap were to reduce the leakage mag-

netism due to the rotor current proportionately to the increase in the magnetising current, the maximum possible power factor would not alter; but this is not what one would expect, and, indeed, from actual tests made by Mr. B. A. Behrend, the effect of enlarging the air-gap of an experimental induction motor was to increase the magnetising current in the ratio of 3 to 7, while the short-circuited rotor current (rotor at rest) only increased in the ratio of 3 to 3·4; thus showing the importance of keeping the air-gap as small as mechanical considerations will admit.

84. **Circle Diagram taking Losses into Account.**—If we take into account the iron losses and the resistance of the stator coils, the diagram Fig. 88 will not be strictly correct; but it will be found that the end, P, of the current vector, O P, still moves upon a circle, as the load on the motor is varied.

Referring to the complete vector diagram Fig. 85 (p. 221), it will be seen that the impressed E.M.F., E, is no longer exactly equal and opposite to the total back E.M.F., E_2; but, owing to the resistance of the primary winding, it is greater than E_2; and the phase angle, θ, between total current, I, and impressed volts, E, is *smaller* than it would be if the primary losses were inappreciable. The other point of difference between Figs. 85 and 87 is that the magnetising component (O I_m) of the primary current, is now more than 90 degrees in advance of the rotor current, O I_2; or, in other words, the angle I I_m O in the triangle O I I_m is something greater than a right angle. We shall, however, assume that this angle—*i.e.*, the phase difference between magnetising and rotor currents—remains constant under all conditions of load, and the two chief differences between the circle diagram Fig. 88 and the

corrected diagram Fig. 89 may be summed up as follows:

1. The vector of the no-load magnetising current (O A), in the corrected diagram, is no longer at right angles to the pressure vector (O E), but makes an angle E O A with the vector O E, which is less than 90 degrees, and corresponds with the actual measured power factor when the motor is running light.

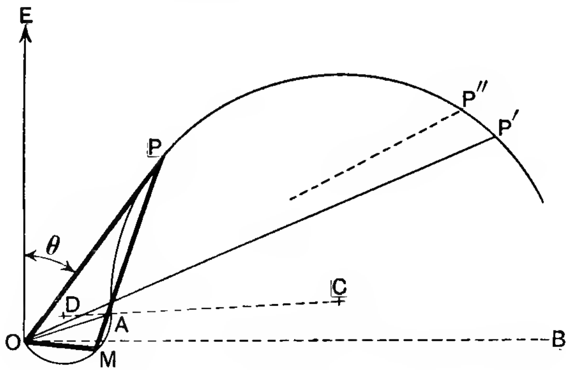

FIG. 89.

2. The angle of phase difference (O M P), between rotor current and magnetising component of primary current,* is somewhat greater than 90 degrees, and will depend upon the iron losses in the motor.

The diagram Fig. 89 has been constructed from measurements taken off Fig. 85, this latter diagram having been re-drawn for various values of the rotor current, and the results all brought to a common basis of comparison by supposing the impressed volts O E to

* Assumed to be of constant value.

remain of a constant value. It will be seen that the centres, C and D, of the two semicircles no longer lie on the line O B, at right angles to O E, but above this line;* otherwise the diagram is very similar to Fig. 88.

The maximum possible value of the primary current will be O P′, which corresponds to the condition of starting (rotor at rest), and the exact position of the point P′ on the circle will depend upon the amount of resistance in the *rotor* windings. The vector O P′ has been drawn on the assumption of a reasonably small rotor resistance—which would result in a loss of pressure somewhat less than that represented by the length O E_r in Fig. 85—but with an increased rotor resistance, the final position of the primary current vector would be on the circle between P′ and P; as, for instance, at P″.

To construct such a diagram which shall represent the behaviour of an actual motor, it is only necessary to have the same particulars as would be required for the construction of a vector diagram of the kind drawn in Fig. 85 (because the circle diagram is merely another form of this general diagram); these particulars are:

1. The no-load current and pressure and true power.

2. The current and pressure and true power with rotor short-circuited, and at rest.

3. The resistance of the primary windings per phase.

It is not proposed to take up any further space here in order to explain how the vector diagram may be drawn from the above particulars; but if the reader cares to follow the matter further, he is referred to Appendix II.

* In Mr. Heyland's original description of his circle diagram (see *Éclairage Électrique*, July 14, 1900), he makes the centre of the large circle fall on the line O B, at right angles to the pressure vector; but this will not be found to give results in strict accordance with the data obtained from actual tests.

at the end of the book. With a few additional lines, the circle diagram can be made to give practically all necessary information regarding the performance of an induction motor by the scaling of lengths, with occasional ratios, or products, of two quantities. Such information would include speed, slip, torque, power losses, and efficiency, for all values of the primary current vector O P. It is not proposed to detract from the simplicity of the diagram Fig. 89 by making the additions referred to; but before leaving the subject of the circle diagram, the main features of Fig. 89 may be summed up as follows:

O E represents the phase of the impressed E.M.F. at primary terminals. The other straight lines are current vectors of which all vertical components (parallel to O E) are "active" currents, while all horizontal components (parallel to O B) are "wattless."

O P, with the large circle as the locus of the point P, is the primary current vector.

Cos θ is the power factor.

P M, which passes through the junction A of the two semicircles, is the secondary current.

O M is the magnetising component of the primary current.

85. **Methods of starting Induction Motors.**—To start an induction motor by merely closing the switches on the supply circuit, and so instantaneously connecting the terminals to the full supply pressure, is evidently unsatisfactory, on account of the very large rush of current which would occur; and, if the motor has to run up under load, this abnormally large current might continue for an appreciable time.

Such a method of starting would only be allowable in the case of very small motors, although it is sometimes

adopted for machines as large as 10 b.h.p. By reducing the pressure across terminals at the moment of switching on, through the introduction of resistances, choking coils, or transformers in the primary circuit, any size of motor, with short-circuited rotor, can conveniently be started up, *provided no great starting torque is required;* that is to say, provided the motor has not to start up against a heavy load.

The method generally adopted for starting motors under load consists in providing the motor with a *wound* rotor, the three sections of the winding having their starting ends connected at a common junction, while their finishing ends are taken to three slip rings, from which the current is collected by brushes in the usual manner. These brushes are connected to the three sections of a variable non-inductive resistance, which may be of the liquid type, or made up of wire spirals joined up to a simple type of controller having, say, half a dozen contacts on each of the three regulating arms. The other ends of the three regulating resistances are all permanently connected together.

When starting up the motor, the controller handle is moved to the position where the three rotor circuits are *open*—*i.e.*, the resistance in series with the windings is infinity. The stator current is then switched on under full pressure, and the controller handle moved forward so as to close the rotor circuit through the resistances, which are gradually cut out as the machine runs up to speed, until, when full speed is attained, all external resistance is cut out, and the rotor windings are left short-circuited upon themselves. These operations are equivalent to inserting a starting resistance in series with the armature of a shunt-wound direct-current motor; a large starting torque can be obtained in this manner, as will

be evident if the reader has carefully followed article 76 and understood the meaning of the curves in Fig. 81.

It has been shown how, by suitably increasing the rotor resistance, the torque, *with rotor at rest*, may be increased up to the maximum torque which the motor can exert under any conditions of working; whereas, if the resistance of the short-circuited rotor is small (which it should be, for the most economical *running* conditions), the torque at starting, without the introduction of some external resistance, will not be equal to the maximum possible torque, although the current taken from the supply mains will be *greater* than that which passes when a starting resistance is inserted. It has also been explained how this effect is due to phase displacement between the rotor currents and the magnetic field upon which they react; and it is, therefore, not proposed to dwell any longer upon this, the most general and satisfactory method of starting.

With regard to the other method already referred to —namely, reducing the supply pressure at starting before connecting the motor to the circuit—perhaps the most approved way of effecting this is by means of auto-transformers.

Fig. 90 shows the connections to a three-phase auto-transformer, such as would be used for a three-phase induction motor up to about 15 b.h.p. size; the arrangement for larger motors being similar, but it is then advisable to arrange for two or more tappings from each of the transformer windings, so that the pressure may be increased gradually, instead of throwing over suddenly from the reduced starting pressure to the full line pressure.

As shown in Fig. 90, the auto-transformer consists of the usual magnetic circuit of laminated iron; but, instead

of there being two distinct sets of coils as in the ordinary transformer, each limb carries only a single continuous winding, with a connection tapped off at a suitable intermediate point. By carefully following the connections on the diagram, it will be seen that, when the three poles

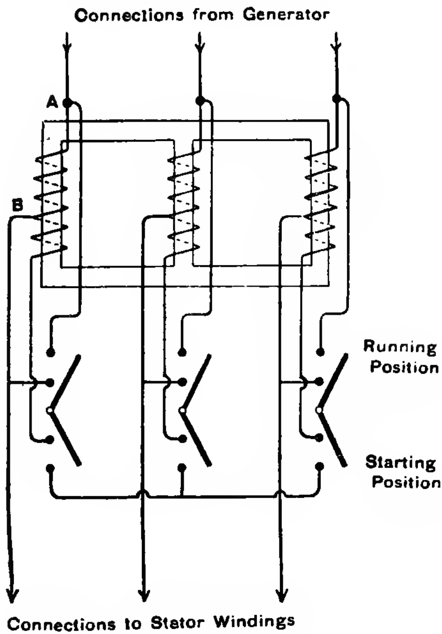

FIG. 90.

of the starting switch are closed on the lower set of contacts, the three transformer windings have their free ends connected to a common "neutral" bar; and they will consequently act merely as choking coils, or as the primaries of a three-phase transformer with open secondary circuit. It will be noticed, however, that the tappings

from somewhere about the centre of the windings go directly to the terminals of the motor, which, instead of receiving the full line pressure, get only a reduced pressure, depending upon the ratio of the number of turns comprised between the points A and B, and the total number of turns on each limb of the magnetic circuit. When the rotor has attained a fairly high speed, the switch is thrown over to the upper, or running position, thus short-circuiting the portion A B of the winding, and leaving the motor terminals connected directly to the supply.

It has already been shown how, if a large starting torque is required, it is necessary to increase the rotor resistance beyond the value which would prove economical, or even possible, for continuous running. Many ingenious methods have been suggested for accomplishing this end without the use of sliding contacts and external resistance coils; but nearly all the methods suggested are costly, even if they do not lead to undesirable complications. The induction motor with "squirrel-cage" rotor is undoubtedly the simplest and the least likely to give trouble when entrusted to unskilled hands, and the method of starting up such a machine by merely closing the switch connecting the stator windings to the supply has much to recommend it. It is possible to start comparatively large motors in this way, if the abnormal rush of current when switching on is not too serious an objection: moreover, for certain conditions, where the highest efficiency is not important, and where heating troubles are not likely to arise, it is possible deliberately to design the rotor with a comparatively high resistance—this being usually provided in the end rings to which the copper conductors are connected.

In conclusion, it has been shown in article 79 that, by keeping up the induction in the iron, the starting torque, for a given arrangement of the windings, will be greater than with low inductions. If the best starting results are to be obtained, particular attention must be paid to the question of magnetic leakage (which must be kept as small as possible), and the resistance of the *stator windings* must be small, otherwise the effective E.M.F. which determines the amount of the magnetic flux may be considerably less than the potential difference at stator terminals.

86. **Speed Regulation.**—The analogy between the induction motor and the shunt-wound direct-current motor, has already been pointed out; both are essentially constant-speed machines, and it is usually uneconomical to use them for variable-speed work. A reduction of the supply voltage does not meet the case, because, although the required back E.M.F. is smaller than with the normal supply voltage across terminals, the magnetic flux through the armature is also reduced, and approximately the same speed is required to produce the (lower) back E.M.F.

There are two ways of regulating the speed of a shunt-wound direct-current motor: (1) Inserting resistance in series with the armature winding, while the full voltage is maintained at the terminals of the field windings; (2) inserting resistance in series with the field windings, while the normal supply voltage is maintained across the brushes. By method (1) the magnetic induction remains constant, but the necessary back E.M.F. will depend upon the value of the resistance in series with the armature. If this is such as to absorb, say, 25 per cent. of the total supply volts, the required back E.M.F. will now be 75 per cent. of what it would be under the

full voltage; the result being that the machine will run at three-quarters of the normal speed. This method is evidently wasteful, since the resistance must necessarily dissipate the whole of the power which is not given out by the motor through the shaft.

By method (2) the magnetic induction through the armature is varied, but the required back E.M.F. remains constant. The speed will therefore vary inversely as the strength of the magnetic field. This method is evidently less wasteful than the first; but, for considerable speed variations, difficulties arise owing to the main field being necessarily weak for the higher speeds, while the leakage field due to the armature currents is relatively large.

Let us now look at the induction motor. Method (1) is almost exactly reproduced by inserting resistance in the *rotor* windings; while the variation in the induction by method (2) would be effected by altering the *frequency* of the supply. (It is the frequency that—for a given impressed E.M.F.—determines the value of the induction; and the *speed* of the rotor, when running light, will therefore vary directly as the frequency.) If a simple form of frequency transformer could be devised, by means of which the frequency of the supply circuit could be gradually altered, this would afford a convenient means of regulating the speed of induction motors, within certain limits, depending upon the amount of the leakage field with the higher frequencies (or low inductions), and the permissible limiting value of the induction in the iron with the lower frequencies and speeds.

By changing the number of stator poles, it is possible to design a motor to run at two or more different speeds, and tests made on such motors have shown the method to be practicable; but it involves some complication in

he stator windings, and the use of special switches with connections so arranged that, by combining the coils in various ways, the number of poles may be altered. Moreover, since an induction motor is usually designed, in the first instance, with as large a number of poles as the diameter of the rotor will allow of, any increase in this number, for the purpose of giving a reduced speed, will lead to a corresponding increase in the magnetic leakage, with its attendant disadvantages.

The series-parallel control of two machines with rotors, coupled mechanically to a common shaft, offers a means of operating efficiently at a speed corresponding to half the frequency of the supply circuit. The method is analogous to the control of two rigidly connected D.C. shunt motors on a constant potential supply, with the armature windings connected in series. The speed of two similar motors so connected will be *half* that of any one machine with full supply pressure across the brushes. The change from half to full speed may be made gradual by connecting the armature windings in parallel through suitable external resistances across the full supply voltage, the resistance being cut out in sections to bring the speed up to the maximum limit. In the case of the induction motor, the secondaries are of the wound type, with slip rings. The primary of motor No. 1 is connected to the supply circuit, but the primary of motor No. 2 is fed by currents from the secondary of No. 1. The *frequency* of the currents in the secondary of an induction motor is proportional to the slip, being zero for zero slip, and equal to the supply frequency when the rotor is at rest. As the speed of the motor set increases, the frequency of the supply to the stator terminals of motor No. 2 will decrease, reaching a limit at half the frequency of the supply circuit. Thus a stable condition of

running is reached at a speed of synchronism, which is just half the speed obtainable with short-circuited rotors and stators supplied with the full line voltage. To effect the change from half to full speed, and obtain intermediate speeds if desired (not without loss of energy during this period), the primary of motor No. 2 is connected directly across the supply mains, while adjustable resistances are inserted in series with the rotor windings. When these resistances are entirely cut out, the speed of the combined set will be the same as the maximum speed of either motor with secondary short-circuited. This method of speed regulation is generally referred to as "tandem" control; but those familiar with the Latin tongue, and desirous of employing a less equivocal term, are at liberty to use the more pertinent word "concatenation."

Returning to method (1) as applied to the speed regulation of induction motors, the effect of introducing resistance in the rotor windings has already been fully discussed (see article 76, p. 205). It must, however, be realised that the reduction in speed is only obtained by wasting the balance of the power in the resistances, exactly as in the case of the direct-current motor with regulating rheostat in series with the armature. A numerical example will make this clear.

Example.—Consider an induction motor of 100 brake horse-power, the efficiency of which is ·9 at full load; and assume that it is required to reduce the speed 20 per cent. The power supplied to the motor at full load is

$$100 \times 746 \times \frac{10}{9} = 83 \text{ kw. (approx.)}$$

and the power absorbed by the resistances inserted in

rotor winding, for a speed reduction of 20 per cent. must be

$$\frac{83 \times 20}{100} = 16{\cdot}6 \text{ kw.}$$

In order to calculate the ohms required per phase, it is necessary to know the value of the full-load current—*i.e.*, the current corresponding to the maximum torque to be exerted by the rotor. Let us assume a star-wound rotor with 300 volts between any two of the three slip rings on open circuit. Then if 80 kw. be taken as the power put into the rotor, the current in each of the three arms of the winding will be

$$I = \frac{80{,}000}{\sqrt{3} \times 300} = 154 \text{ amperes.}$$

Now, since the total power dissipated in the resistance is 16·6 kw., the watts absorbed by each of the three sections will be

$$\frac{16{,}600}{3} = 5{,}530.$$

Hence $I^2 R \text{ (per phase)} = 5{,}530,$

which gives us for the resistance of each section of the resistance

$$R = \frac{5{,}530}{(154)^2} = {\cdot}233 \text{ ohms.}$$

If, instead of absorbing the difference of pressure in a resistance, it were possible to insert a back E.M.F. in the rotor windings, from an outside source, exactly the same result would be obtained. Such a method has, indeed, been suggested; but since some form of com-

mutator is required, involving the use of a rotor wound somewhat in the same manner as a direct-current motor armature, the machine becomes more costly, and loses its peculiar advantage of mechanical simplicity owing to the absence of a commutator.

87. **Reversing Direction of Revolution.**—The direction in which a polyphase induction motor will run is determined by the direction of rotation of the magnetic field. By reversing the direction in which the field revolves, the direction of running of the rotor will also be reversed. In fact, a simple form of throw-over switch in the connections to the stator terminals is generally all that is necessary.

If the supply is three-phase, it matters not whether the coils are star or mesh connected, but, provided the stator is fed by only three wires, the effect of crossing the connections to any two of the terminals will cause the reversal of the rotating field, and, therefore, of the rotor.

88. **Recapitulation.**—We have now studied the induction motor by means of vector diagrams, and seen how this type of machine may be considered as a special case of the alternate-current transformer, the points of difference being:

1. That the secondary winding is free to revolve in the field produced by the primary, and so generate a second E.M.F. called the E.M.F. of rotation which—by acting against the induced E.M.F.—limits the flow of current until this is just sufficient to produce the required torque.

2. Owing to the necessary mechanical clearance, or air-gap between primary and secondary cores, the magnetising current is greater than in the ordinary closed-circuit transformer; moreover, the total primary current, under light load conditions, is more nearly in

phase with the true (or "wattless") magnetising component, because the "energy" component due to the hysteresis and eddy-current losses is comparatively small.

3. The magnetic leakage is much greater than in a well-designed static transformer. It is the aim of the designer to keep this as small as possible, because a large leakage flux means small overload capacity, although when the machine is running lightly loaded—*i.e., when the rotor currents are small*—nearly all the magnetic flux passes through the rotor, and the leakage is in any case small.

This leads up to the consideration of what is the best shape of slot through which the stator windings are threaded. In this country, and on the Continent, completely closed slots with an exceedingly thin metal bridge adjoining the air-gap are not unusual. In America a small gap at the top of the slot is generally provided, the object being to reduce the magnetic leakage and avoid the high labour cost of winding. This has little effect upon the running qualities of the motor (except as regards the overload capacity); but it admits of a *greater starting torque* being obtained, or the same starting torque with a reduced primary current, owing to the fact that a larger proportion of the total primary flux passes through the rotor. It must, however, not be overlooked that open slots, although they may reduce the leakage, generally increase the reluctance of the magnetic circuit through the rotor, and consequently the amount of the "wattless" magnetising current. They also lead to "pulsation" of the flux, owing to want of uniformity in the air-gap flux distribution, an effect which is practically absent when closed slots are used, and which is largely overcome by using partially closed slots, as the "fringing" with these slots makes them almost equivalent to closed slots, in

that they produce no appreciable flux pulsation in the teeth. Thus higher flux densities may be used in closed slot machines than in open slot machines, and where the cost of winding is not excessive, closed slot machines will be the cheaper as well as the lighter for the same output and speed. For the same weight and size, the closed slot machine will generally have the better characteristics.

In conclusion, although the writer prefers to consider the polyphase induction motor as a modified transformer with a closed secondary free to revolve in two alternating magnetic fields, differing in *direction* by 90 "electrical space-degrees," and with a *phase* difference of a quarter period, it does not follow that it is incorrect or more difficult to treat the subject from the "rotating field" point of view. Indeed, if the reader will again turn to article 31, Chapter III., he will probably find that his conception of a short-circuited rotor being dragged round by the revolving field is clearer to him than before reading the present chapter.

CHAPTER VIII

ASYNCHRONOUS GENERATORS, FREQUENCY CONVERTERS, COMPENSATED INDUCTION MOTORS, AND ROTARY CONVERTERS

89. **The Polyphase Induction Motor used as a Generator.**—It has been explained in the last chapter how, when the load is put on an induction motor, the "slip," or the *difference in speed between rotor and magnetic field* increases, the reason being that, with a demand for increased torque, a greater rotor current is required to meet it, and—with an approximately constant speed and strength of the rotating field—this increased current can only be obtained by a lowering of the back E.M.F. of rotation in the rotor conductors such as will be the immediate result of increased "slip."

When the motor is running light, the slip is small—that is to say, the rotor is running nearly, but not quite, at synchronous speed—this being due to the fact that a small rotor current must necessarily flow, or sufficient torque will not be produced to overcome bearing friction, windage, etc.

Imagine a small auxiliary motor coupled to the shaft, and supplied with power from an outside source. This motor can be arranged to increase the speed of the rotor until this is exactly equal to the speed of synchronism: the rotor current will then be zero, but the auxiliary

motor will be supplying the power to overcome the light-load losses referred to above.

Suppose, now, that the speed of the auxiliary motor be increased so as to drive the rotor of the polyphase machine at a speed slightly *above synchronism—i.e., greater than that of the rotating field.* The result will be that the E.M.F. generated in the rotor conductors, due to the cutting of the magnetic lines, will now be *greater* than the induced E.M.F.; and currents will, therefore, flow in the rotor windings in a direction exactly opposite to that obtained under ordinary conditions of working. The *negative* rotor current necessarily implies a negative primary current; or, in other words, the component of the total primary current which balances the rotor current will now be forced to *flow back into the supply circuit, against the primary impressed E.M.F.;* and the induction motor—when mechanically driven at a speed above synchronism—becomes a *generator of electric power.*

Such machines are known as asynchronous generators, to distinguish them from the more usual form of synchronous alternator or polyphase generator. It should be particularly noted that the periodicity of the currents obtained from the terminals of an asynchronous generator is *independent of the rotor speed,* being determined solely by the frequency of the E.M.F. supply brought to the terminals, for the purpose of producing the magnetic field, and without which these machines would not work.

90. **Vector Diagrams of Asynchronous Generator.**—In Fig. 91 the vector diagram of an induction motor has been drawn to illustrate the effect of mechanically driving the rotor at a speed slightly above synchronism, and so transforming the machine into a generator of electric power. This diagram should be compared with Fig. 82 on p. 209, and it will be seen

how the difference lies in the rotor current, I_2, being drawn exactly equal to the current I_2 in Fig. 82, but *in an opposite direction.* The same assumptions have been made in order to simplify the diagram, these being (1) that the resistance of stator windings is negligible, and (2) that the rotor windings are without self-induction. This last assumption involves the idea of the *whole of the*

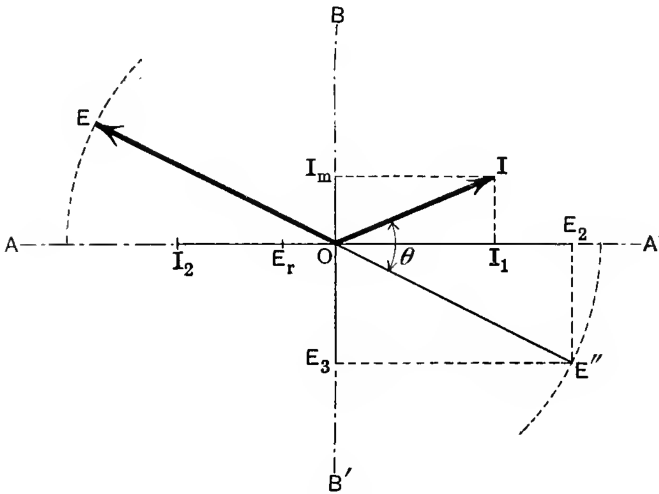

FIG. 91.

leakage magnetism enclosing the *stator windings only*, and *not passing through the iron core on which the rotor conductors are wound.* The current vector, OI_m, is, therefore, the magnetising component of that portion of the total magnetic flux which passes through both rotor and stator windings. It induces an E.M.F., E_2, in the rotor coils; and, since these coils are being revolved at a *speed above synchronism*, the E.M.F., E_2 E_r, due to the rotation in the

flux at right angles to the inducing flux, is now *greater* than the induced E.M.F., O E_2, the result being a current, O I_2, flowing *against* the induced E.M.F. The balancing component of the stator current must, of course, be O I_1, exactly equal but opposite to O I_2: the E.M.F. due to the leakage field will be O E_3, which must now be drawn *below* the line A A′ instead of above, as in Fig. 82; and, by completing the diagram, we obtain O E as the total necessary impressed E.M.F. corresponding to the current, O I_2, flowing in the rotor.

We now see how the product of the vectors O E and O I is *negative*, since the current in stator windings is flowing against the impressed E.M.F., and therefore returning power to the circuit. All the vectors are of the same length as in Fig. 82, and the angles θ are equal. It follows that the power given back to the circuit, according to Fig. 91, is the same as that which was supplied to the motor according to Fig. 82, and the power supplying the I^2 R losses in the rotor—represented by the product O I_2 × O E_r—is taken from the driving source, and transmitted to the rotor through the shaft.

In Fig. 92 the asynchronous machine, A, and the synchronous machine, Z, are shown coupled electrically on a load which may consist of incandescent lamps or induction motors, or both. The connections are shown for one phase only, so as not to complicate the diagram. The current through the stator coils of machine A is I, corresponding to the vector O I in Fig. 91. This may be considered as made up of two components—the current, L, going to the load, and the current, S, passing through the armature windings of the synchronous machine, Z.

We shall suppose the condition of things to be as represented by Fig. 91—that is to say, the synchronous

machine, Z, runs at a constant speed, and produces a definite pressure, E, at terminals, while the machine A runs at a constant speed somewhat above the speed of synchronism, which results in a definite value of the current O I and a definite phase angle θ between current and impressed E.M.F.

In Fig. 93 the diagram Fig. 91 has been re-drawn in order to show merely the current O I making an angle θ with the pressure vector, O E″, which may be considered as the E.M.F. *at the terminals of the load*, while O E represents the E.M.F. generated by the synchronous

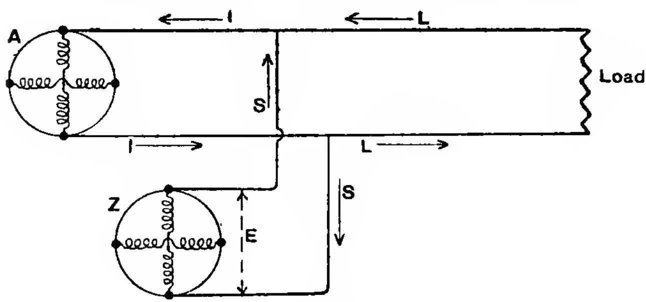

FIG. 92.

machine. Drop a perpendicular, I L_1, on to O E″, and note that, *if the load is non-inductive* and equal to the product O E″ × O I cos θ, then the asynchronous machine, A, is taking the whole of the load, while the synchronous machine, Z, acts merely the part of an exciter, and does no work, since the current component, O S_1, is "wattless." If the load is greater than this amount, and equal to O E″ × O L_2, then machine A will still do its share, and give the whole of its output to the load; but the balance required (which is equal to O E × L_2 L_1) must

be supplied by the machine Z, the total output of which may be written (O E) × (L_2 I) cos β, where L_2 I is the current S in the diagram Fig. 92.

If the external load is *less* than the full output of the asynchronous machine, as, for instance, O E″ × O L_3, then the current, S, in the armature of the synchronous machine is represented by the vector L_3 I, which makes an angle with O E greater than 90 degrees. It follows that the power given out by machine Z is negative—*i.e.*,

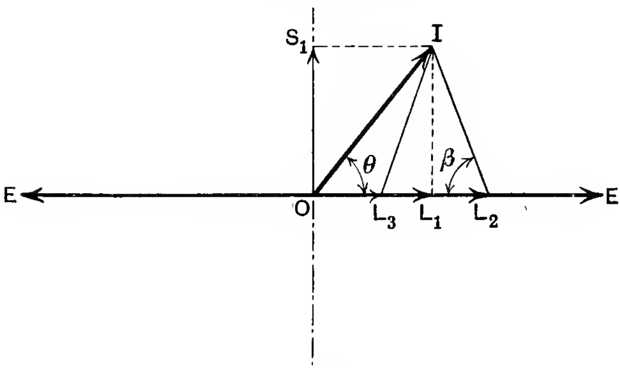

FIG. 93.

this machine is being *driven as a motor* by the machine A, the power expended in doing this being equal to O E″ × L_3 L_1.

Let us now see what is the effect of an inductive load. The diagram Fig. 94 has been drawn in a similar manner to Fig. 93, but the load current now lags behind the pressure O E″, by a certain angle ϕ. If the load current is O L_1, such that I L_1 is at right angles to O E″, then the *power* absorbed is the same as before, being equal to O E″ × O P or to O E″ × O I cos θ, which is the total

output of the asynchronous machine—for the particular conditions as regards speed and excitation that we have assumed. But the "wattless" current, O S_1, from the armature of the synchronous machine is now greater than O S_1 in Fig. 93 by an amount P L_1, which is exactly equal to the wattless component of the load current; from which it is clear that any "wattless" currents required, owing to the load having self-induction, cannot be provided by the asynchronous machine, but must be

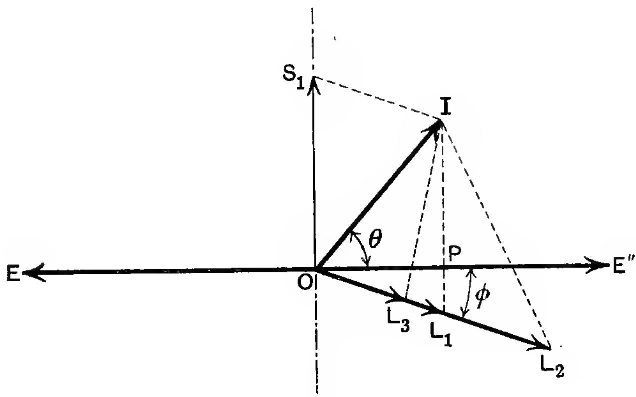

FIG. 94.

supplied by the synchronous generator *in addition to the wattless exciting current* required by the former. If O L_2 is the load current, L_2 I is the current S (Fig. 92) from machine *Z*; while, if the load current is O L_3, the power absorbed is less than the full output of the machine A, and the synchronous machine is again being driven as a motor.

91. **Conclusions regarding the Induction Motor used as a Generator.**—Since the frequency is inde-

pendent of the speed of an asynchronous generator (being determined solely by the speed of the synchronous machine which supplies the magnetising currents to the stator winding), it follows that machines of this class can be paralleled without the necessity of regulating the speed to such a nicety as when paralleling the more common type of alternating-current generators. In fact, the various units have merely to be run up to approximately synchronous speed, and switched on to the 'bus bars, much in the same manner as when coupling direct-current dynamos. The proportion of the total load taken up by each machine (where several are coupled in parallel) will depend solely upon the relative *speeds* at which the various units are driven.

It would almost seem as if this convenience in paralleling were the only recommendation for machines of this class.

They have the disadvantage of requiring a synchronous generator, always running, to fix the frequency and provide the wattless magnetising currents; moreover, since the asynchronous machines are capable of supplying the external circuit only with *energy* currents, the synchronous generator may have to be of a very large size if the load is inductive, seeing that it will be called upon to supply the wattless component of the total load current, in addition to the magnetising currents of the other generators.

With the advent of the compensated induction motor as introduced and developed by Mr. Alex. Heyland and others, there is a possibility of such machines—when used as generators—proving a commercial success, owing mainly to the fact that they can be loaded inductively, thus bringing the size of the synchronous exciter within reasonable limits. It is not proposed to discuss this

question at greater length; but since there is a possibility—even if little probability—of the compensated polyphase induction motor with commutator being used extensively in the near future, the theory and properties of such motors will be briefly dealt with in article 93.

There is one point in connection with the asynchronous generator which should, perhaps, be mentioned, and this is the peculiar effect of a heavy overload. If the speed of the prime mover be increased beyond a certain limit, the proportion of the load taken by the generator *will begin to fall off*, and unless suitable precautions are taken, there is a possibility of the engine "running away." This will be evident from a study of Fig. 91, which, as already pointed out, differs but little from the corresponding diagram of the same machine used as a motor. In the case of the motor it has been shown how, when the torque exceeds a certain limit, the leakage field becomes so great that the machine is unable to respond to the additional call upon it, and falls out of step. In the generator the effect is similar, and the overload capacity of the machine is reached when the speed exceeds the speed of synchronism by an amount approximately equal to the "slip" corresponding to the maximum output of the same machine when run as a motor.

92. **Frequency Converters.**—Although the frequency of the currents in the primary circuit of an induction motor remains unaltered whatever may be the speed of the rotor, the frequency of the currents in the secondary windings is dependent upon the speed of rotation. Thus, when the rotor is at rest, the machine may be looked upon as a static transformer of inferior design, with rather poor regulation because of the increased leakage due to the air-gap and necessary separation between primary and secondary windings; the periodicity of the

secondary E.M.F. will, however, be the same as that of the primary impressed E.M.F. If, now, a certain speed of rotation be imparted to the rotor by means of an independent motor coupled to the shaft, the E.M.F. in the secondary windings will depend upon the relative speed of rotor and revolving field. In this connection, a mental picture of the revolving field is perhaps more useful than the recently-discussed equivalent of two flux components with an angular displacement of 90 electrical (space) degrees. If the rotor is provided with slip-rings, the frequency of the current taken from the collecting brushes will be directly proportional to the *difference* in speed between revolving field and rotor. If the rotor is driven *backward—i.e.*, in a direction contrary to that of the rotating field—the secondary frequency will be *higher* than that of the primary circuit.

In order to obtain a constant frequency at secondary terminals, the *slip* (whether positive or negative) must bear a definite relation to the primary frequency, and the motor driving the shaft must therefore be a constant speed machine, such as a synchronous motor connected to the primary supply circuit. The ratio of frequencies is obviously

$$\frac{\text{secondary frequency}}{\text{primary frequency}} = \frac{\text{“ slip ” revolutions}}{\text{synchronous speed}}.$$

In regard to the power capacity of the auxiliary motor, when the slip is 100 per cent.—*i.e.*, when the rotor is at a standstill—the auxiliary motor is doing no work; and when the slip is 200 per cent.—*i.e.*, when the rotor is driven *backward* at a speed corresponding to that of the revolving field—the auxiliary motor is providing the same power as the primary circuit supplies through transformer action. This should be evident without proof;

but, bearing in mind that the total output plus the losses must exactly equal the joint capacity of the auxiliary driving motor and the converter proper, and further that *torque* × *slip* represents the power demanded of the auxiliary motor, it will be seen that the ratio of primary to secondary frequency is also the ratio of "transformer power" to total power, the difference between these quantities being provided by the auxiliary driving motor.

As a frequency changer *per se*, the induction motor with independently driven rotor, as above described, is rarely met with in connection with practical undertakings; but when combined with the rotary converter, it forms the input unit of the motor converter, a type of machine that will be referred to in article 100.

93. **"Compensated" Polyphase Motors with Commutators.**—To Mr. Alex. Heyland must be given credit for the idea of supplying the magnetising currents of an induction motor through the rotor windings instead of through the stator coils. This ingenious method has the effect of improving the power factor, which can thus be brought up to about 100 per cent. for all loads. Other workers, including Messrs. Gorges, Latour, and Osnos, have devised machines somewhat on the same lines; but since there is no difference in the principle involved, we shall content ourselves with briefly considering how, by feeding the magnetising currents through a commutator into the rotor windings, it is possible to compensate for phase displacement, and bring the power factor, even at light loads, approximately up to unity.

In Fig. 95, the rotor is supposed to be of the wound type, and provided with a commutator in all respects similar to that of a direct-current bipolar machine.

Imagine the rotating field to be produced by two alternating fields, one acting in the direction B B′, and the other—differing in phase by a quarter period—acting in the direction A A′, at right angles to B B′; this being the method of treating the subject which was justified

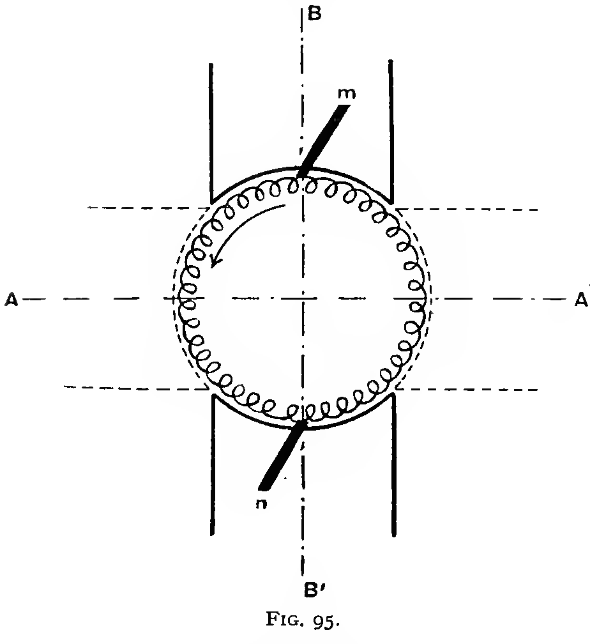

FIG. 95.

and explained in the last chapter, when describing th action of the ordinary induction motor.

Consider, first, what goes on in the phase B B′ only. The stator windings, in this phase, act merely as the primary of a static transformer; and, if we suppose the rotor or secondary windings to be without any short-

circuiting connections, the current that will flow in the stator windings when these are connected to the supply mains will be the magnetising current; and although the *true* magnetising watts—*i.e.*, the product of the current and *resultant* volts—may be small, the *apparent* watts are relatively great, since they are equal to the product of the current and the full pressure of the supply.

Suppose now that, with the armature at rest, and with the stator windings disconnected from the supply, an alternating current of the same frequency, but suitably reduced pressure, is fed into the brushes, *m* and *n*, bearing on the commutator at the two ends of a diameter lying on the magnetic axis, B B′, as shown in Fig 95. Th current will divide itself between the two halves of the winding, and produce an alternating field in the direction B B′, which—if the pressure be adjusted to suit the number of turns in the rotor winding—may be made of the same strength as the field originally produced by the stator winding. But *with the rotor at rest* the power factor has not been improved, since the rotor coils may now be considered as constituting the primary winding of a transformer, and the same trouble occurs—*i.e.*, the magnetising current produces the magnetic flux, which, in its turn, gives rise to a back E.M.F. of self-induction very much in excess of the resultant E.M.F. required to overcome the resistance of the windings, and this necessitates an impressed E.M.F. not only slightly greater than the induced E.M.F., but—on account of the relatively high reluctance of the air gaps—nearly 90 degrees in advance of the current, as regards phase.

Now imagine the rotor to be revolving at synchronous speed. The brushes bear on the commutator, but retain a fixed position in space, the result being that the alternating current fed into the brushes will produce an

alternating flux in the same direction, and of the same frequency as before; *this frequency being quite independent of the speed at which the rotor is revolving*, and depending merely upon the frequency of the supply connected across the brushes. It is true that the frequency of the current in *any particular portion of the rotor winding* will depend upon the speed of revolution—and, in fact, for the condition of synchronous speed that we are considering, the current in the rotor conductors would be a continuous one; but the point which concerns us at present is that, as the rotor revolves, the conductors lying between the brushes, *m* and *n*, are cutting the field A A′ at such a rate as to produce an alternating E.M.F. of rotation exactly equal, but opposite in direction, to the alternating E.M.F. of induction due to the field B B′. No further difficulty need therefore be experienced in getting the magnetising current through the rotor windings. The E.M.F. across the brushes, *m* and *n*, no longer requires to be of an abnormally great value, since the induced back E.M.F. is now balanced by the E.M.F. of rotation. A few volts only are needed to overcome the ohmic resistance of the windings, and provide the necessary magnetising current. Moreover, this pressure will now be *in phase with the magnetising current*, and not nearly 90 degrees in advance, as was necessary so long as the back E.M.F. of self-induction was a factor to be reckoned with. It follows that the magnetising watts are now *true watts*, and, by regulating the pressure across the brushes, *m* and *n*, until the magnetising ampere turns are equal to those which otherwise would have to be provided by the stator current, the latter may be reduced to nearly zero, thus bringing the power factor (even with the machine running light) to a value very near to unity.

When the motor is loaded, the magnetising current

requires to be of approximately the same value as on open circuit, and the power factor can be maintained at nearly 100 per cent. for all loads. It is evident that, as the load comes on the motor, the speed is no longer that of synchronism; but it does not differ much from the speed of the rotating field, and at full load, with a "slip" of, say, 3 per cent., the currents in the rotor conductors, although no longer direct currents, are of such a low periodicity that self-induction troubles do not arise.

This is the theory of the compensated induction motor; it will be understood that the exact pressure across the brushes should be adjusted experimentally for each particular machine, to suit the supply voltage. If the pressure is too low, the exciting current in the rotor windings will be insufficient to provide the full amount of the flux, and it will, therefore, be supplemented by a small magnetising component of the stator current, which means that the power factor will still be less than unity. If too great a current is put into the rotor windings, this will give rise to a *leading* component of the primary current of such a value as to neutralise the excess of magnetising current in the rotor, and maintain the resultant magnetising ampere turns at the exact value required to produce the necessary back E.M.F. in the stator windings.

Although it is not proposed to enter into details of construction, or to discuss the various devices by means of which this theory of compensation may be applied in practice, this article would hardly be complete without a reference to the actual method of compensating induction motors originally devised by Mr. Heyland.

In Fig. 96, the stator windings of the motor are shown star-connected off the supply mains. The three rotor coils, C C C, are also star-connected, being short-circuited

at their outer ends by the heavy connection, S, while their inner ends are joined through the *low resistance*, R. This resistance actually takes the form of small V-shaped

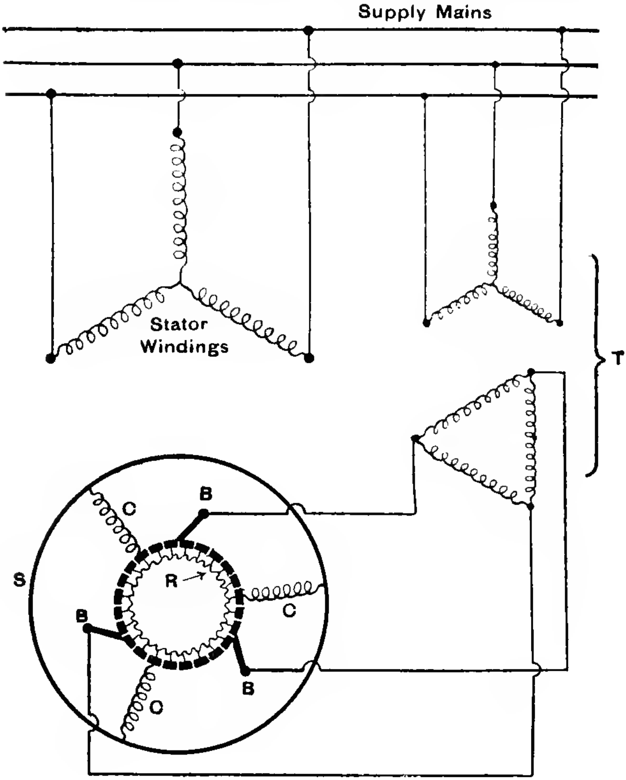

FIG. 96.

connectors of low-resistance material joining contiguous segments of the commutator; and the currents that are brought to the brushes, B B B, will circulate in the rotor

coils, C, in such a direction as to produce a rotary magnetic field synchronous with the field that would otherwise be produced by the magnetising components of the stator currents. The magnetising currents led into the brushes are obtained from the secondary of a small transformer, T, of which the primary is connected to the same source of supply as the stator windings. It will be seen that the primary coils of this transformer are star-connected, while the secondary coils are mesh-connected; the object of this being to provide currents to the brushes, B, which, instead of being in phase with the supply pressure, are 90 degrees out of phase, and, therefore, of the same phase as the required magnetic flux.

It is not necessary to use the same winding for the magnetising currents as for the circulation of the currents which arise when the load is put on the machine, and in the Heyland type of motor of later design a double three-phase winding is used on the rotor with advantage.

Such machines when used as asynchronous generators have undoubted advantages over the ordinary type of machine, and it will be readily understood that there are no difficulties in the way of compounding these compensated generators, since the necessary currents for strengthening the field can be taken off suitable current transformers, and led into the commutator through brushes in the same manner as the compensating currents.

Although the compensated and compounded polyphase induction motor is undoubtedly interesting and instructive from the scientific point of view, it is doubtful whether it has come to stay, because unless the advantage of an improved power factor is worth the increased complication and expense of manufacture, the machine can never prove a commercial success.

It must not be overlooked that the addition of a commutator, with all the increased difficulties of winding, is out of the question for small machines; and when we have to deal with large machines, the power factor of these at full load is usually over 90 per cent., and in order to raise this to 100 per cent. it is a doubtful point whether the increased cost and increased liability to break down are justified.

94. **Synchronous Rotary Converters.**—The synchronous converter is a machine provided with a commutator, which receives alternating currents and delivers continuous currents all through the one set of armature coils. It is generally used for transforming polyphase into direct currents, the polyphase currents being led into the armature windings through slip rings, while the direct current is collected by brushes bearing on the commutator. The machine may, therefore, be considered as a combined synchronous motor and direct-current generator.

When treating of polyphase generators, it was shown how the sides of a closed polygon of vectors represent the respective alternating E.M.F.s in the various (mesh-connected) armature coils. Thus, in Fig. 97 (*a*), the vectors A B, B C, and C A, forming the sides of an equilateral triangle, represent, by their magnitude and direction, the amount and phase relations of the E.M.F.s in the three windings of a delta-connected three-phase generator or motor.

In Fig. 97 (*b*) and 97 (*c*), the vector diagrams have been drawn for four- and five-phase machines respectively, while the dotted hexagon in Fig. 97 (*a*) refers to a six-phase machine. The fact that all these figures are *closed* polygons is an indication that the sum of all E.M.F.s acting in the armature winding is zero, and

consequently currents, such as may properly be represented by vector diagrams, will not circulate in the coils when the external circuit is open.

Now, since A B, B C, etc., represent the alternating

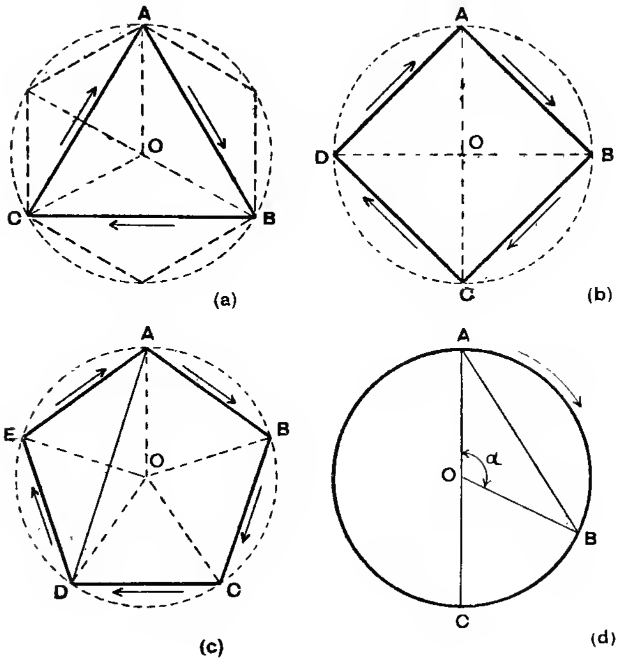

FIG. 97.

E.M.F.s in the various sections of the winding, which are all connected in series, the pressure between any two terminals of the polyphase circuit (which is equal to the sum of the component E.M.F.s) will be indicated on the vector diagrams, both in magnitude and phase, by the

straight line joining the beginning of the first and the end of the last vector of which the sum is taken. Thus, in Fig. 97 (*b*), the pressure between opposite terminals is indicated by the lines A C and B D, the length of which is a measure of the E.M.F. obtained by connecting two of the phases in series: the *phase* of the resultant E.M.F. is also indicated by the angular position of these lines relatively to the component vectors; but the direction—*i.e.*, whether, for instance, from A to C or C to A—cannot be shown by an arrow on the diagram, as it will depend upon what is to be considered as the positive direction in the external circuit joining the two terminals.

In Fig. 97 (*c*), the pressure between the terminals A and D will be represented by the length of the line A D, which may be looked upon as the sum of the three vectors A B, B C, and C D, or of the two vectors D E and E A, for these must necessarily balance, and give the same resultant.

Let us now carry this construction further, by increasing the number of the sides of the polygon, and therefore the number of phases, indefinitely. This leads us to the diagram Fig. 97 (*d*), which represents the E.M.F. in the armature winding of the machine as being due to the rotation, in the field, of an infinite number of armature sections, in each one of which an alternating E.M.F. is generated. The circle may be considered as the limit of the polygon of vectors as the sides of the polygon are increased in number without limit; and the length of the line A B represents the alternating pressure obtained between two tappings from the winding separated by a distance equivalent to the phase angle α. With regard to the diameter A C, this represents the pressure obtained between two diametrically opposed points of the (two-pole)

armature winding; and if we suppose the E.M.F. wave to follow the simple harmonic law of variation, the direct-current pressure between the brushes bearing on the commutator of a rotary converter or double-current generator would be $\sqrt{2}$ times the value given by the length A C.*

The diagrams, as drawn in Fig. 97, are not limited in their application to two-pole machines; it is merely necessary to bear in mind that the angles in the diagrams are *phase angles*, and the vector A C in Fig. 97 (*b*) or (*d*) indicates the pressure between two points on the actual winding separated by a distance equal to the pitch of the poles: thus, it is only in the case of a two-pole machine that these tappings would actually be at the two ends of a diameter.

A rotary converter can evidently not be wound for any desired pressure transformation; the ratio of alternating to direct current pressure is fixed, being determined solely by the number of slip rings or phases. From an inspection of the diagrams in Fig. 97, it will be seen that, since the length of the chord joining two points on the circle is a measure of the alternating E.M.F. between

* If the E.M.F. vectors such as A B and A C stand for the *maximum* values of the alternating pressures instead of the $\sqrt{\text{mean square}}$ values, then the diameter of the circle would correctly indicate the direct-current voltage between brushes, no multiplier being necessary. It is evident that, when tappings are taken from diametrically opposed points on the winding, these points must simultaneously pass under the direct-current collecting brushes twice during each revolution of the armature, and at these moments the instantaneous value of the alternating E.M.F. must necessarily correspond with the direct-current voltage: moreover, since the brushes lie on the neutral axis, where the E.M.F. is at its maximum, it follows that it is the *maximum* value of the alternating E.M.F. which agrees with the direct-current voltage.

these points, the maximum value of this E.M.F. is given by the expression

$$E_{\text{max.}} = 2 \cdot R \sin \frac{\alpha}{2},$$

where R is the radius of the circle and α is the phase angle between the two tappings off the armature winding. Expressed in terms of the direct-current voltage, we have

$$E_{\text{max.}} = D \sin \frac{\alpha}{2};$$

where D—the diameter of the circle—stands for the direct-current pressure across brushes. If we assume the sine curve wave form, the $\sqrt{\text{mean square}}$ value of the alternating pressure will be equal to the above value divided by $\sqrt{2}$.

The following table gives the pressure per phase for various numbers of slip rings, the direct-current pressure being taken as unity:

Number of Phases or Slip Rings.	$\text{Sin} \frac{\alpha}{2}$	E.M.F. between Slip Rings = $\text{Sin} \frac{\alpha}{2} \div \sqrt{2}$
Single phase or two phase	1	·7071
Three phase	$\frac{\sqrt{3}}{2} = \cdot 866$	·6124
Four phase	$\frac{1}{\sqrt{2}} = \cdot 7071$	·5000
Five phase	·5878	·4157
Six phase	$\frac{1}{2} = \cdot 5000$	·3536
Twelve phase	·2588	·1830

These calculated values agree fairly well with figures obtained from actual machines.

95. **Elementary Theory of the Rotary Converter.**—Consider a rotary converter running at full speed with the direct-current circuit open—*i.e.*, without any direct current being taken out of the armature. The condition is merely that of a synchronous alternating-current or polyphase motor running light. Let us suppose the field coils to be excited from an independent direct-current source, and note that—when the armature is revolving at synchronous speed—there is a definite strength of field required to produce the necessary back E.M.F. of rotation in the armature conductors. If the current in the field coils is adjusted to give this particular strength of field, the alternating currents fed into the armature through the slip rings from the supply mains will be very small, being merely such as to produce the torque required to overcome the light load losses; moreover, this current will be in phase with the applied E.M.F.

If we now either increase or decrease the field current, the result will be an alteration in the armature current, which must adjust itself in order that the resultant magnetising ampere turns may be the same as before: thus, if the current in the field coils be strengthened, the armature current will *lead* the applied E.M.F., while if the excitation is reduced, the current in the armature will be a lagging one.*

Once the idea of a constant resultant field excitation is clear, it is an easy matter to follow the actions which take place when a direct current is taken out of the

* Incidentally, attention may be called to the possibility of regulating the power factor of a polyphase system by over or under exciting the field coils of rotary converters connected to the mains. These machines will, in this respect, act in a similar manner to synchronous motors, and they may be made to produce the effect of either condensers or choking coils as the case may require.

armature through the brushes bearing on the commutator. Whatever portion of the direct current passes through sections of the armature winding must be balanced at every instant by currents taken from the polyphase supply mains, in order that the resultant field excitation shall not be altered. It is true that the direct current taken from the armature tends mainly, if not wholly, to produce cross magnetising ampere turns; but these cross magnetising turns must nevertheless be balanced by the polyphase current entering the armature through the slip rings, because—even on the assumption of an efficiency of 100 per cent.—whatever power is taken out on the direct-current side must evidently be balanced by an equal power supplied to the armature from the polyphase side.

On account of this balancing of the magnetising effects due to the continuous and polyphase currents, there is practically no armature reaction in the polyphase rotary converter. If the losses are negligible, and the field current is adjusted to give the maximum power factor, there is theoretically very little or no armature reaction, and this accounts for the fact that it is not necessary to alter the position of the brushes with varying load. In this respect the rotary converter differs from the double-current generator, because, in the latter, both direct and alternating currents combine to produce field distortion.*

* The double-current generator is a machine from the armature of which both alternating and continuous currents may be taken. It has not been considered in previous chapters, because it is rarely used. It is neither so satisfactory nor so useful a machine as the synchronous converter; but, so far as the component parts and general appearance are concerned, it may be looked upon as a synchronous converter with the addition of a shaft coupling or pulley through which it receives mechanical power from an outside source.

It is well known that, in the continuous-current transformer with two windings and two commutators, there is no trouble with sparking at the brushes, owing to the fact that the cross magnetising ampere turns of the secondary winding are balanced by the ampere turns of the primary winding; and although, in the polyphase rotary converter, there is only one winding, and the current taken out of the machine is not of the same kind as the current put in, yet, at every instant, there is a balancing action similar to that which occurs in the continuous-current transformer.

At certain moments during the revolution of the armature, the current will pass direct from slip ring to commutator brush, and at all other times the transfer of energy may be considered as due either to direct conduction through the windings between slip rings and brushes, or to balanced generator and motor action—certain portions of the winding acting as generator while the other portions act as motor. There is an exception in the case of the single-phase rotary converter, the action of which is slightly more complicated; here some of the energy (about one-third of the total) must be considered as transferred by *unbalanced* generator and motor action, which involves the idea of a flywheel effect, and a certain storage of energy in the revolving armature during portions of a complete period, this energy being given back again in the form of direct currents at other instants. This leads, in some cases, to sparking, and it is one of the reasons why the single-phase rotary converter is not so satisfactory a machine as the polyphase converter.

96. **Output and Efficiency of Rotary Converters.** —Without entering fully into the question of I_2 R losses in the armature windings of rotary converters, it should be understood that the output of these machines—*except*

in the case of the single-phase converter—is greater than that of the same machine used merely as a direct-current generator; that is to say, the $I_2 R$ losses are appreciably *less* than if the whole of the output were passed through the windings in the form of a direct current. That this must be the case is fairly obvious, especially when it is remembered that, as the connections from the slip rings pass under the brushes, there is a direct transfer of current from slip ring to brush *which does not pass through the armature conductors.*

The output of a rotary converter will also be increased with the number of phases or slip rings. Thus, the $I_2 R$ losses in a machine provided with three slip rings, and fed by a three-phase current, will be greater than if the same machine is provided with six slip rings and fed by a six-phase current. This, indeed, is what would be expected, seeing that the increased number of phases reduces the current *per phase* required to balance the continuous current. For instance, if we assume a constant direct-current output of 100 kw. at 100 volts and a power factor of unity on the polyphase side, the current per phase in each armature section for a total input of 100 kw. (all losses neglected) would be 544 amperes for a three-phase supply, and only 472 amperes for a six-phase supply. The table on p. 273, based on particulars given in Dr. A. S. McAllister's book on "Alternating-Current Motors," gives the approximate output of a rotary converter with different numbers of slip rings; the output of the same machine rated as a direct-current generator being taken as unity. The power factor is assumed to be 100 per cent.

If the power factor is less than unity, the output will be less; thus, if $\cos \theta = \cdot 8$ the output of the single-phase converter would be only $\cdot 7$, while that of the three-phase

and four-phase machines would be about 1·15 and 1·4 respectively.

Another advantage resulting from an increase of the number of phases beyond three is that the heating of the armature conductors is more evenly distributed. The $I^2 R$ losses are not evenly distributed throughout the armature windings of a rotary converter; but with an increase in the number of phases, there will not only be less local heating, but the $I^2 R$ losses will be dissipated more uniformly throughout the entire armature.

Since the output of a rotary converter increases with

Number of Slip Rings or Phases.	Output of Rotary Converter.
Two (single phase)	·85
Three	1·35
Four (two or four phase)	1·65
Five	1·80
Six	1·95
Eight	2·05
Ten	2·15
Infinity	2·30

the number of slip rings or phases on the alternating-current side, it is obvious that, if an economical and simple means can be employed for transforming a two or three phase supply into a polyphase system having a larger number of phases, this may be advantageous under certain conditions. The problem, indeed, is not a difficult one, as many solutions can be obtained by a suitable combination of vectors differing in phase. Fig. 98 illustrates one method, known as the "double delta," by means of which a three-phase supply can readily be transformed into a six-phase system suitable for feeding a rotary converter provided with six slip rings. The

upper diagram shows the primaries of the three step-down transformers mesh-connected between the supply mains. Each transformer is wound with two equal but distinct secondaries, and the six sections of secondary winding thus obtained are connected up in such a manner as to produce a six-phase supply. The combi-

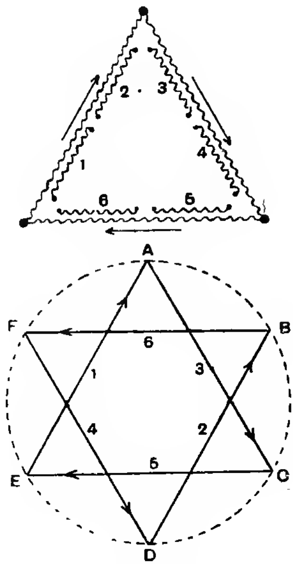

FIG. 98.

nation of vectors is shown in the lower diagram, and it will be seen that the desired result is obtained by mesh-connecting the coils 1, 3, and 5 in one direction, and the coils 2, 6, and 4 in the opposite direction. By feeding the slip rings from the junctions A, B, C, D, E, and F, the six-phase supply to the armature is obtained.

Messrs. C. P. Steinmetz and L. R. Emmet have devised an equivalent arrangement of static transformers, very much on the lines of Mr. Scott's phase-transforming system, by means of which the same end is attained. This is known as the "double tee" method.

Mr. A. D. Lunt has shown how a twelve-phase system can be obtained in a similar manner from a three-phase supply, but the supposed advantages of increasing the number of phases to this extent in a synchronous converter are not obvious. Even in the case of very large rotaries, the disadvantages of having twelve slip rings, with all the increased complication and expense, would seem to outweigh the possible advantage of slightly improved efficiency or increased output. The theoretical output of the twelve-phase rotary is only about 12 per cent. greater than that of a six-phase machine.

The efficiency of rotary converters compares favourably with that of motor-generator sets: it would be about 90 per cent., at full load, for a three-phase 200-kw. machine at a periodicity of 40, or for a 400-kw. machine at a periodicity of 60, while the efficiency of an equivalent motor-generator set would not be much above 86 per cent. It should be understood that these figures include the losses in the step-down transformers required for use with the rotary converter; but the motor of the motor-generator set would be wound for the high-tension supply, which might be of the order of 6,000 volts.

The higher frequency machines are not so satisfactory as those built for the lower frequencies. It is well to remember that the synchronous converter is really a compromise between a direct-current machine and a polyphase generator or alternator, as the case may be. There is no objection to building an alternating-current generator with a sufficient number of poles to give a

frequency of 60 cycles; but the most satisfactory design of large direct coupled D.C. generator would have fewer poles for the same output, the frequency of currents in armature coils being of the order of 8 to 12; while in the case of high-speed, belt-driven machines the frequency may well be as high as 20. If, therefore, the rotary converter is designed for a frequency of 25 (which is usual in America), the number of poles is somewhat in excess of what would be considered good practice in a continuous-current generator, and the synchronous converters of 50 and 60 cycles are necessarily more difficult to design, and usually of less satisfactory performance.

Tests have been made, and figures published, tending to show that, as regards the all-day efficiency of rotaries and motor-generators, there is not much to choose between the two; but the cost of the motor-generator is generally somewhat higher than that of the equivalent rotary converter, although, when the necessary step-down transformers and regulators are taken into account, the difference in this respect is not so great as it is sometimes supposed to be.

97. **Starting and Synchronising Rotary Converters.**—When possible, it is desirable to start up a rotary converter from the direct-current side: this is a simple matter, and the operation is in all respects similar to the method adopted for starting up an ordinary shunt-wound direct-current motor. The machine is then synchronised by varying the field excitation until the correct speed—as indicated by the synchroniser—is obtained, when the switches on the polyphase side can be closed. If the supply is three-phase, it is usual to have one of the switches closed, and, at the moment when synchronism is attained, the other two switches are thrown in. The operation of synchronising has to be carefully carried

out, or trouble may ensue. In cases of emergency, the process of synchronising may be omitted; the machine should be run up to a speed slightly above synchronism, when the *direct-current switches must be opened*, and the switches on the polyphase side immediately closed, thus connecting the armature to the polyphase supply. The machine will pull itself into step, and the direct-current switches can then again be closed.

When it is not possible to start up from the direct-current side, a very satisfactory method consists in providing a small induction motor for the purpose of running the armature up to speed. This auxiliary motor is designed to run at a slightly higher speed than the rotary, and the latter is then brought down to synchronous speed by loading one of the phases through a suitable rheostat. This method makes the process of synchronising exceedingly simple, and, as the power developed by the induction motor need only be about one-tenth of the full load output of the rotary, the extra cost is not very great.

There is another method of starting up from the polyphase side which, however, is only applicable to small machines, and this consists in switching the polyphase supply directly on to the machine, having previously taken the precaution to disconnect the field windings and divide them into several sections: the main switches on the direct-current side must, of course, also be open. The revolving field due to the polyphase currents in the armature causes the machine to start up much in the same way as an induction motor; and when synchronism is reached—as indicated by the steady reading of a voltmeter across the direct-current brushes—the field circuit may be closed; but certain precautions must be taken to ensure that the field circuit is closed on the phase which

will give the right polarity across the direct-current brushes. The object of dividing up the field winding into several distinct sections is to prevent the accumulated E.M.F. induced in these windings during the period of running up, reaching such a high value as to risk a breakdown of insulation. One objection to this method of starting is that a very large current is taken from the supply mains, amounting to, perhaps, three times the normal full-load current of the rotary, and since this current is very much out of phase with the supply pressure, the regulation of the polyphase system will be seriously affected. It is true that, before switching on the supply, provision can be made for reducing the voltage across slip rings by taking intermediate tappings from the low-tension side of the step-down transformers, but the extra cables and switch gear required are a somewhat serious objection.

98. **Regulation of Rotary Converters.**—Since the direct-current pressure across brushes has a definite value for a given alternating pressure across slip rings, it follows that the regulation cannot be effected, as in the case of a motor-generator, by merely altering the strength of the field current. Any alteration in the current passing through the field windings is immediately met by an alteration in the armature current, which so adjusts itself as to leave the resultant field as before—*i.e.*, of such a value as to produce the required back E.M.F. corresponding to the speed of synchronism.

If the pressure of the polyphase supply is raised or lowered, then the direct-current pressure will be varied in a corresponding manner. This suggests what is, perhaps, the most perfect method of regulation : namely, the employment of variable ratio step-down transformers, by means of which—even with constant pressure at the

high-tension terminals of the polyphase supply—the volts across slip rings can be varied within the required limits. This regulation can be done by hand or automatically; but automatic regulation on these lines would be costly and liable to give trouble owing to complication of parts.

Another method of varying the pressure across slip rings, which is automatic in its action, is of considerable interest; and since it is largely used in practice, it is advisable that the principles involved be clearly understood. In the first place, this method requires the compound winding of the field magnets; but, from what has already been stated, it is evident that the mere strengthening of the field ampere turns as the load increases will not produce the required pressure rise across slip rings to compensate for the volts lost in overcoming the resistance of the windings, *except under special conditions* which permit of this result being attained. These conditions are that there must be an appreciable amount of self-induction in the circuit between the high-tension supply terminals (at constant pressure) and the slip rings of the rotary, and that the ohmic resistance of this portion of the circuit must be relatively small. If the step-down transformers are of a type having a fair amount of magnetic leakage, it may not be necessary to provide additional self-induction in the form of choking coils; but, in practice, small choking coils in each of the polyphase leads are usually inserted.

In Fig. 99, the full line curve shows the relation between armature current and field excitation for a rotary converter or synchronous motor running light. The ordinates, or vertical measurements, indicate the amount of the armature current, while the horizontal measurements represent the ampere turns on the field magnets. There is a certain value of the field current—denoted on

the diagram by the letter S—for which the armature current is a minimum; the power factor is then approximately unity, and the very small armature current passing into the windings is merely what is necessary to run the machine light. If, now, we imagine the field current

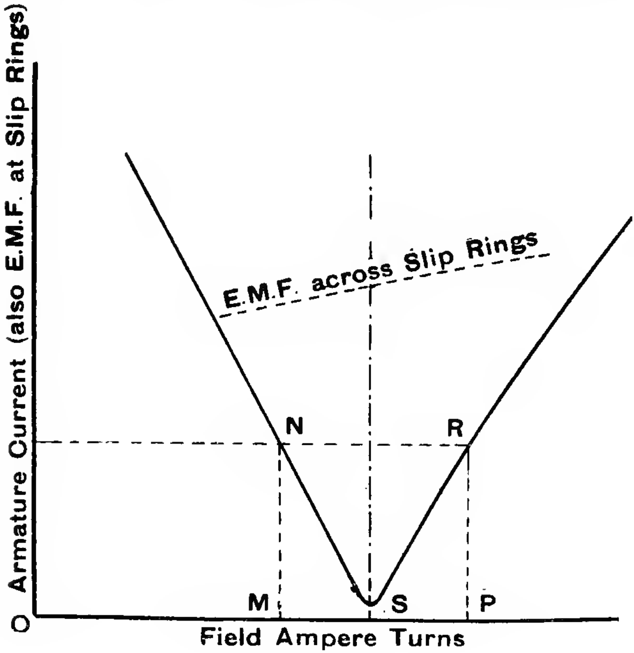

Fig. 99.

to be reduced to a value indicated by the distance O M in Fig. 99, the armature current will immediately rise to the corresponding value M N, and this will be *a lagging* current, the principal component of which is a magnetising current 90 degrees behind the impressed E.M.F.,

because this will produce a flux in the same direction as the flux due to the direct current in the magnet coils, and its value will be such as to provide the magnetising ampere turns which have been taken off the field coils. If, on the other hand, we strengthen the field current, the main component of the armature current will *lead* the impressed E.M.F. by 90 degrees, and so counteract the magnetising force of the additional field ampere turns.

For a given value, O P, of the field current, the no-load

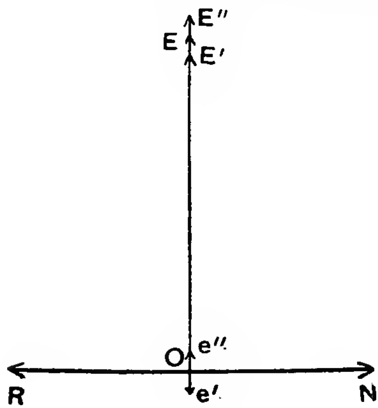

FIG. 100.

armature current may be of exactly the same value as when the field current was O M; but in the first case it will be *in advance* of the impressed E.M.F. by nearly a quarter period; while in the latter case it will *lag behind* the impressed E.M.F. by the same phase angle.

Now consider the vector diagram Fig. 100. Here O E represents one phase of the impressed polyphase E.M.F., which is supposed to be of constant value. We shall not complicate the diagram by taking into account the step-down transformers, and the length O E

must be considered as representing the primary pressure *reduced to the pressure at slip rings*, in accordance with the transformer ratios, for the condition of the correct field excitation—*i.e.*, when the field current is such as to make the power factor a maximum (O S in Fig. 99). Under this condition the armature current is practically zero Let us now see what will be the pressure at slip rings with a reduced field excitation and, therefore, a lagging armature current, which may be represented by the vector O N.*

Note, first, that, *if there were no self-induction in the circuit*, the pressure across slip rings would remain unaltered (except for the small resistance drop, which may be neglected); but if the circuit has self-induction, the back E.M.F. due to the current O N will be O e', drawn exactly 90 degrees behind O N. The result will be that the pressure at slip rings will be reduced to O E′, which is less than O E by the amount O e'. Suppose now that the field current is increased until the armature current is a leading one of value O R; then O e'' will be the induced E.M.F. due to this current, and it will be in phase with O E: the pressure at slip rings will, therefore, be O E″, which is equal to O E + O e''.

We are now in a position to plot the dotted curve

* In this diagram (Fig. 100) the current vectors are drawn exactly 90 degrees out of phase with the pressure vector, although in practice this condition is never quite fulfilled. But, with the machine running light, the phase angle will be very nearly 90 degrees, and, in any case, even if the machine were loaded, it would be correct, for the purpose of this argument, to consider O N as the magnetising component of the total armature current, because the "active" component, in phase with O E, will produce only cross magnetising effects, and, even in the choking coils, will not give rise to a back E.M.F. having any appreciable effect on the regulation.

in Fig 99, showing how the pressure at slip rings increases with increased field ampere turns, and from this it is easy to understand that a rotary converter provided with a shunt winding across the brushes, and a series winding carrying the load current, may—by suitably adjusting the amount of self-induction—be made practically self-regulating at all loads.

It is important to bear in mind that, in order to obtain this result, the ohmic resistance of the cables, transformer windings, etc., must be low; and this is of still greater importance if it is desired to *over-compound* the rotaries. If the distance of transmission is great, or the voltage so low as to involve a considerable drop of pressure in the cables, then this method of regulation is unsuitable.

The chief objection to the compound winding of rotary converters as above described is that the power factor of the polyphase supply is continually changing, and this makes it difficult to maintain constant pressure at the receiving ends of the transmission lines. Mr. A. C. Eborall, in his paper on polyphase substation machinery,* states that, in practice, the self-induction in the polyphase leads and the amount of the field excitation would be so adjusted as to make the armature current at no load a lagging one equal to about 30 or 40 per cent. of the full-load current. As the load comes on the machine, the power factor improves and becomes nearly unity at half full load, after which, with a further increase of load (and, therefore, of field ampere turns), the armature current would be in advance of the applied E.M.F.

99. **"Hunting" of Rotary Converters.**—The satisfactory running of synchronous motors, and, therefore,

* "Some Notes on Polyphase Substation Machinery," *Journ. Inst. E. E.*, vol. xxx., p. 702.

also of rotary converters, depends largely upon the engines in the generating station. If these have not sufficient flywheel effect, or are otherwise defective in the matter of imparting a uniform angular velocity to the generators, then trouble is to be looked for in connection with synchronous machines taking current from the supply. By uniformity of angular velocity is meant constant speed *per revolution*; and a high-speed engine will necessarily be more satisfactory in this respect than a low-speed engine: it also follows that "phase swinging" or "hunting" troubles are not likely to arise when the generators are driven by steam or water turbines.

Sometimes rotary converters will "hunt" even when the engines are in all respects suitable for the work they have to do. A slight oscillation may be started through various causes, and—especially if there are several machines working in parallel—this may tend to increase, owing to periodic distortion of the field produced by the "swinging" of the armature, and the resulting variations in the phase angle of the current fed into the slip rings. It is, therefore, advisable to design the rotaries to check, as far as possible, this tendency to "phase swinging" which may result in the machines falling out of synchronism, not to mention the probability of severe sparking at the commutator brushes.

Damping coils or "amortisseurs" suggest themselves at once as a suitable means of preventing distortion of the magnetic field under the pole faces; and these coils may take the form of extra heavy high-conductivity metal flanges supporting the field coils at the ends nearest to the pole faces. The use of such castings, or of heavy short-circuited copper conductors between the pole shoes, or imbedded in the pole face, may lower

the efficiency from 1 to $1\frac{1}{2}$ per cent.; but this is of no consequence compared with the importance of steady running and freedom from hunting troubles. As a matter of fact, solid cast-iron pole faces act in a similar manner to the damping coils, and are equally effective.

Hunting is far less likely to occur with low than with high periodicities, and 60 may be considered as the upper limit, beyond which it would be inadvisable to work rotary converters: sparking troubles may be expected, and—in any case for large units—motor-generators will generally be found preferable at this, or higher periodicities. The best frequency for rotary converters lies between 25 and 40. The machines work well on lower frequencies, but the cost of step-down transformers becomes excessive.

Hunting troubles will be increased if the connections on the polyphase side are of high resistance, and, for this reason, rotary converters are generally found to work better when within a comparatively short distance of the generating plant than when placed at the distant end of fairly long transmission cables.

In conclusion, it should be stated that, although the rotary converter has been considered throughout as taking the supply from the polyphase mains (which is the usual arrangement), the machine is reversible, and can be supplied with direct current at the brushes, in which case it will give out alternating currents at the slip rings. If such a machine be used for supplying alternating or polyphase currents in parallel with other machines, difficulties arise owing to the absence of armature reaction. Any lagging currents on the alternating-current side tend to weaken the field, and generally produce a condition of instability, requiring special

methods of field excitation to remedy the trouble and prevent the rotary falling out of step.

100. **Motor Converters.**—The La Cour Motor Converter consists of two machines with a common shaft. The input machine (No. 1) is an induction motor with three slip rings for starting purposes, and a wound rotor which feeds current into the armature of machine No. 2. The output machine is a synchronous converter provided with a commutator, all as described in article 94. The

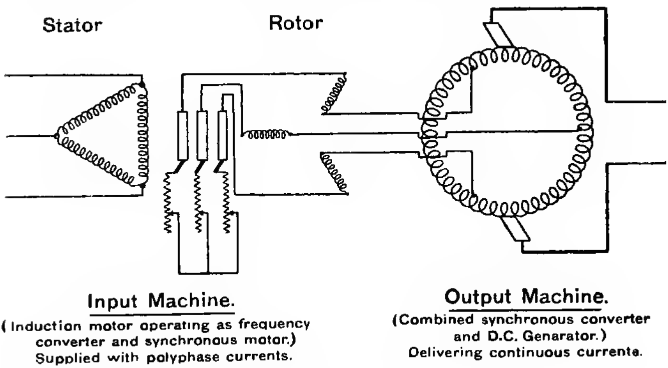

FIG. 101.

diagram of connections, Fig. 101, will serve to explain the principle of action of the motor converter. This diagram shows the rotor of the input machine wound for only three phases, and feeding the armature of the output machine through connecting wires (carried along or inside the common shaft) terminating at points on the armature 120 electrical space-degrees apart. The diagram would appear to indicate that the output unit (the synchronous converter) is a two-pole machine; but it can obviously be designed with any suitable

number of poles; and, further, since the synchronous converter is a more efficient and satisfactory machine as the number of phases of the supply current is increased, it is usually supplied with six or twelve phase currents in the practical motor converter. The increase in the number of phases is readily accomplished without appreciable increase in cost, since it is merely necessary to take the requisite number of taps off the rotor of No. 1 machine, and connect these to the corresponding points on the armature of No. 2 machine. It may be mentioned that, when the rotor of the induction motor is wound to give, say, twelve-phase currents, it is not necessary to provide more than three slip rings for starting purposes: the motor is started up by gradually cutting out the external resistance from the three phases only; but all twelve windings must be finally short-circuited when full speed has been attained. Bearing these points in mind, we may now briefly consider the action of the machine on the basis of the diagram Fig. 101.

If the input and output units have the same number of poles, the speed at which the combined set will operate will be exactly half the speed of synchronism of the input machine considered as an ordinary induction motor with short-circuited rotor coils. The reason of this will be clear when it is considered that No. 2 unit, acting as a synchronous converter, is fed with currents having a frequency proportional to the *slip* of the rotor of No. 1 unit. With both machines at rest, this frequency is the same as that impressed on the stator windings of machine No. 1; but as the speed of rotation increases, this frequency decreases until a condition of balance is obtained, when the speed will remain constant for a given impressed primary fre-

quency. The exact relations between speed and frequency and relative number of poles are best explained with the aid of symbols.

Let f_2 = frequency impressed on armature of machine No. 2;
p_2 = number of pairs of field poles of machine No. 2;
R = actual speed of revolution of both machines (revolutions per second);
f_1 = frequency impressed on stator windings of machine No. 1;
p_1 = number of pairs of poles on No. 1 machine;
R_1 = synchronous speed (revolutions per second) of rotor of No. 1 machine considered as induction motor with secondary windings short-circuited:

then,
$$f_2 = R \times p_2\,;$$
$$f_1 = R_1 \times p_1\,;$$

$$\text{but } f_2 = f_1 \times \frac{\text{slip revolutions}}{R_1} = f_1 \times \frac{R_1 - R}{R_1}.$$

Substituting this value of f_2 in the first equation, we get

$$f_1 = \frac{R\,R_1}{(R_1 - R)} \times p_2,$$

and by substituting this value of f_1 in the second equation we eliminate the frequencies, and get the relation

$$p_1 = p_2 \times \frac{R}{(R_1 - R)},$$

which can be put in the form

$$R = \frac{p_1\,R_1}{(p_1 + p_2)},$$

or $$R = \frac{f_1}{(p_1 + p_2)}.$$

Thus, the combined set of two machines has a definite speed which is independent of the load, and bears a definite relation to the impressed frequency and the number of poles on the input and output units. When $p_1 = p_2$ the speed is obviously exactly half that of the revolving field of the input machine, as already mentioned.

The action of the two machines connected together in the manner above described may be summed up as follows: When continuous currents are taken from the brushes of the output unit, the power is supplied by the input unit in two ways—(1) by *transformer action*, the input machine being considered as a frequency converter; and (2) by *motor action*, the input machine being considered as a *synchronous motor* transmitting mechanical power through the common shaft. The output machine, on the other hand, may be regarded as a combination of a synchronous converter and a direct-current generator. A little study will show that the ratio *power transmitted mechanically* to *power transmitted by transformer action* is the same as the ratio p_1 to p_2.

The chief advantage of the motor converter over the synchronous converter lies in the fact that the commutating machine can be supplied with polyphase currents of a frequency considerably below that of the primary supply circuit. When low primary frequencies are used, this advantage no longer exists; but the motor converter is a satisfactory and efficient machine, with the starting characteristics of the ordinary polyphase motor. This makes the starting up of the motor converter a very simple matter.

APPENDIX I

On the Relation between Magnetic Flux and Induced E.M.F. in a Circuit conveying an Alternating Current

Let N be the total amount of magnetic flux through a coil of wire due to a certain definite maximum value of the alternating current. We need not concern ourselves here with the relation between the current and the total flux; but this can be approximately predetermined in the usual way (as in dynamo and motor design), provided the material, arrangement, and measurements of the magnetic circuit are known.

Let S be the number of turns in the coil of wire, and f the periodicity of the current (number of cycles per second); then, since the total number of magnetic lines denoted by N are twice created and twice withdrawn during the course of one complete period, the *mean* value of the induced E.M.F. will be

$$E^{\circ}_a = 4\,N\,S\,f,$$

and this equation is true whatever may be the shape of the current wave.

In the above formula E°_a is the induced E.M.F. expressed in absolute C.G.S. units. If we wish to express the mean induced E.M.F., E_a, in *volts*, we must write

$$E_a = \frac{4\,N\,S\,f}{100{,}000{,}000}$$

It is not, however, the *mean* value of the induced E.M.F. which we generally require to know, but its $\sqrt{\text{mean square}}$ value. Let us denote the ratio $\dfrac{\sqrt{\text{mean square volts}}}{\text{mean volts}}$ by the letter m, which therefore stands for the quantity usually referred to as the *form factor:* we may now write,

$$\text{Induced E.M.F. (in volts)} = \text{E} = \frac{4m\,f\,\text{N}\,\text{S}}{10^8}.$$

If the E.M.F. wave is a sine curve, the form factor is $m = \dfrac{\pi}{2\sqrt{2}} = 1{\cdot}11$, and $\text{E} = \dfrac{2\,\pi\,f\,\text{N}\,\text{S}}{\sqrt{2} \times 10^8}$.

Consider now a circuit in which the flux N is directly proportional to the strength of the current I which produces it, as, for instance, a solenoid or coil without iron core. The constant *coefficient of self-induction* can be expressed in terms of N, S, and I, its value being

$$\text{L} = \frac{\text{flux linkages}}{\text{current}},$$

which, expressed in *henrys*, is

$$\text{L} = \frac{\text{N}\,\text{S}}{10^8 \times \text{I}_m},$$

where I_m is the maximum value of the current wave corresponding to the production of the maximum number of magnetic lines N.

On the sine wave assumption, $\text{I}_m = \sqrt{2}\,\text{I}$, where I is the R.M.S. value of the current; and the above formula can be written

$$\text{N}\,\text{S} = 10^8\,\text{L}\,\text{I}\,\sqrt{2}.$$

If this value for the total flux linkages be inserted in the voltage equation, we get

$$E = 2 \pi f L I,$$

which is the well-known formula for reactance voltage as used in connection with the analytical solution of alternating-current problems.

APPENDIX II

Method of Drawing the Complete Vector Diagram for a Polyphase Induction Motor from Three Sets of Measurements made on the Machine

The three sets of measurements required, in order that all conditions of working may be predetermined, are as follows:

(1) The current, volts, and true watts per phase of the stator winding when the motor is running light, at practically synchronous speed.

(2) The current, volts, and true power per phase of the stator winding, with rotor short-circuited and held in fixed position to prevent turning.

(3) The resistance per phase of the stator winding.

The set of readings (1) should be made with the full working voltage on primary terminals; but for the set of readings (2) the volts at primary terminals should be reduced, to allow of a current not exceeding the normal full-load current to pass: in both cases the frequency must be the same as that for which the motor is designed, and on which it will have to work.

The general diagram, Fig. 85, which appeared on p. 221, has been reproduced here, together with the new

diagram Fig. 85A, which explains the construction from the above data.

Draw O A (Fig. 85A) in any direction, to represent the magnetising current when motor is running light; and O B, to represent the applied potential difference at stator terminals; the angle between these two vectors

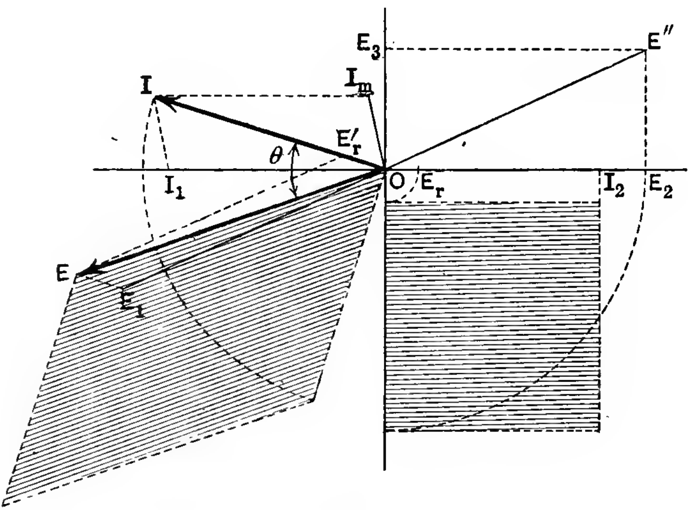

FIG. 85.

being α, which must be such that $\cos \alpha =$ the power factor, or ratio of true watts to apparent watts.

Now draw B H parallel to O A, and of such a length as to represent, to the same scale as O B, the volts lost in overcoming the ohmic resistance of the stator windings: it will be equal to (O A) × R, where R stands for the resistance per phase of the primary winding, referred to under test (3). Join H O and produce to K,

making O K equal to O H. This gives us the vector O K for the E.M.F. of self-induction (which, in the rotor, is balanced by the E.M.F. of rotation in the magnetic field; with the result that there is no appreciable current in the rotor conductors). It is this pressure vector which—in Fig. 85—is supposed to be of constant value. We can, therefore, draw the dotted circles, of radius O K or O H, from the centre, O, and—under all conditions of load—the vector representing the total induced E.M.F. per phase of the primary circuit must

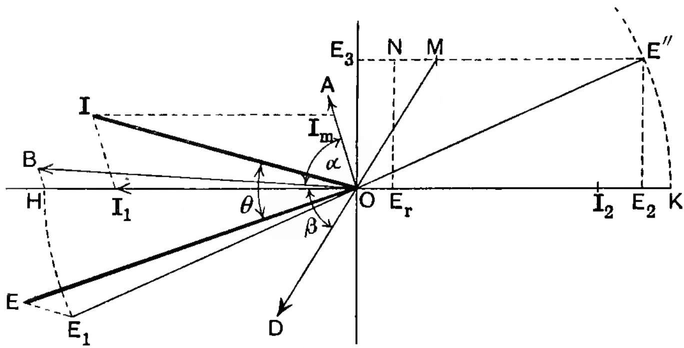

FIG. 85A.

lie on the arc of circle described through K, while its balancing E.M.F. component of the total primary pressure will lie on the arc of circle described through H.

We will now turn our attention to the second set of measurements—*i.e.*, the terminal volts and the true watts corresponding to a definite primary current I_1, obtained when the rotor is short-circuited and clamped in position to prevent rotation.

Note, in the first place, that the magnetising current (in the phase O A) will be so small as to be negligible,

being merely such as will induce in the rotor the very low volts, E_r, required to overcome the resistance of the windings. It follows that the total primary current, I_1, is almost exactly equal and opposite to the rotor current, I_2, and these current vectors must, therefore, be drawn respectively in the phases O H and O K.

Now draw O D in advance of O I_1, to represent the measured potential difference per phase at primary terminals, the angle β between the two vectors being such that $\cos \beta =$ the ratio of (measured) true watts to apparent watts, and produce D O to M, making O M = O D.

The vector O D may be considered as being made up of two components at right angles; one of these, in phase with the current, and equal to M E_3, is required to overcome the resistance of stator and rotor windings, while the other (equal but opposite to O E_3) is required to balance the E.M.F. of self-induction due to the leakage magnetism which does not enter the rotor. In this manner we obtain the length of the vector O E_3, corresponding to a given secondary current, I_2; and for any other value of the rotor current, this leakage E.M.F. vector may be assumed to vary directly as the current.

Referring again to the vector E_3 M, this is made up of two parts, M N and E_3 N, representing the E.M.F. components required to overcome primary and secondary resistances respectively. It is easy to calculate M N, since it is equal to (O I_1) × R, where R = primary resistance per phase,* and this leaves E_3 N,

* With a mesh-connected winding it is the "equivalent" star resistance that should be taken. The exact meaning of this quantity is discussed in a note on stator winding resistance measurements, contributed by the writer to the *Journal of Electricity*, of San Francisco, May 9, 1914.

or O E_r for the E.M.F. required to overcome the secondary or rotor resistance. (It is interesting to note that we have, in this way, arrived at the equivalent resistance of the rotor conductors without making a direct measurement of same.)

In order to complete the diagram corresponding to the working conditions when the rotor current is I_2, produce E_3 M (parallel to O K) until it meets the dotted circle at E″: drop the perpendicular E″ E_2 on to O K, and produce E″ O to E_1, making O E_1 equal to O E″.

The vector O E_2 is the induced E.M.F. in rotor under working conditions, and it will require a magnetising current O I_m to produce it, of such a value that O I_m bears the same relation to O A as O E_2 bears to O K. This enables us to obtain the primary current vector O I. Now draw E_1 E parallel to O I, and equal to (O I) × R, where R = primary resistance. Join O E: then O I and O E are the vectors representing the primary current and potential difference at terminals which will be required, under working conditions, when the rotor current has the value I_2; the power factor being evidently cos θ, where θ is the angle subtended by these vectors.

In this manner the complete diagram similar to Fig. 85 can be constructed from the three sets of measurements referred to above; and it is evidently only necessary to carry out the construction for one more value of the rotor current (equal to about 2 I_2)—no additional tests being required for this purpose—in order to obtain three points on the circle diagram, which can then be constructed in all respects as shown in Fig. 89 on p. 232.

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